Empa
Materials Science and Technology

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Laser Processing of Materials

Optical Properties of Materials and Light-Material Interaction

Patrik Hoffmann

Contents

- Laser light - materials interaction
  - dielectric constants, refractive index
  - absorption / absorption mechanisms
  - spectroscopy
  - refraction/reflection
  - properties of dielectrics
  - properties of metals
  - scattering

Remarks

- Recommended literature:

Optical properties of solids / Mark Fox. 2nd ed.. Oxford : Oxford University Press ; 2010

Definitions

E(r,t) [$V \cdot m^{-1}$] – electric field

D(r,t) [$C \cdot m^{-2}$] – electric displacement

H(r,t) [$A \cdot m^{-1}$] – magnetic field

B(r,t) [T] – magnetic induction

P(r,t) – polarisation

ε – relative permittivity

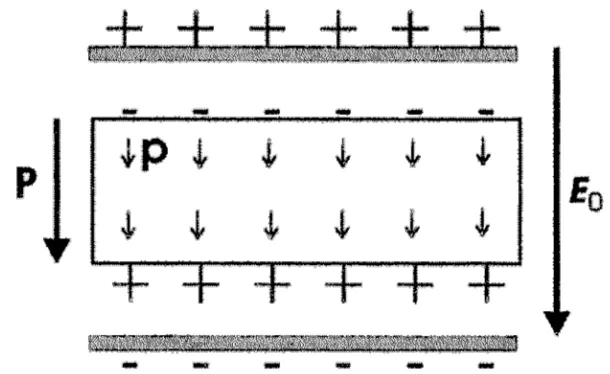
χ – electric susceptibility

μ - relative permeability

χ_M – magnetic susceptibility

ε_0 – permittivity of vacuum

μ_0 - permeability or vacuum



Electric material response:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 (1+\chi)\vec{E} = \varepsilon_0 \varepsilon \vec{E}$$

$$\vec{P} = \varepsilon_0 \chi \vec{E}$$

$$\varepsilon = 1 + \chi$$

Magnetic material response:

$$\vec{B} = \mu_0 \vec{H} + \vec{M} = \mu_0 (1+\chi_M)\vec{H} = \mu_0 \mu \vec{H}$$

$$\vec{M} = \mu_0 \chi_M \vec{H}$$

$$\mu = 1 + \chi_M$$

Maxwell's Equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ Faraday's law

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$ Ampere's law

$$\nabla \cdot \vec{D} = \rho$$ Coulomb's law

$$\nabla \cdot \vec{B} = 0$$ absence of magnetic charge

E(r,t) [$V \cdot m^{-1}$] – electric field

D(r,t) [$C \cdot m^{-2}$] – electric displacement

H(r,t) [$A \cdot m^{-1}$] – magnetic field

B(r,t) [T] – magnetic induction

J(r,t) - current

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$ conservation of electric charge

Wave Propagation

for light (EM-wave) propagation normally a simplified set of Maxwell's equations can be used, since currents and charges are not present

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{D} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\vec{J} \equiv 0$$

$$\rho \equiv 0$$

description of a plane wave can be, for example, used to find solutions of Maxwell's equations:

$$\vec{E}(z,t) = \vec{E}_0 e^{i(k \cdot z - \omega \cdot t)}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi n}{\lambda_0} = \frac{n \cdot \omega}{c} - \text{wave vector (photon impulse)}$$

$$\omega = 2\pi\nu - \text{light frequency (photon energy)}$$

Photon Energy Units

depending on region of the EM-spectrum and discussed application, different units can be used

$$E[J] = eE[eV] = \hbar \cdot \omega[Hz] = h \cdot \nu[Hz] = \frac{\hbar \cdot c}{n} k[cm^{-1}] = \frac{h \cdot c}{\lambda_0[nm, \mu m]}$$

this is for convenience and/or by tradition
all units can be converted to another one

n - refractive index

Refractive index come out as a coefficient of Maxwell’s equations for propagation of EM-wave

$$v_{EM-wave} = \frac{1}{\sqrt{\varepsilon\mu}}c$$

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

$$n = \sqrt{\varepsilon \cdot \mu}$$

ε - relative permittivity
μ - relative permeability

$$n = \sqrt{\varepsilon}$$

at ω optical frequencies - μ~1

If absorption is present:

$$\tilde{n}^2 = (n + ik)^2 = \varepsilon' + i\varepsilon''$$

$$n^2 - k^2 = \varepsilon'$$

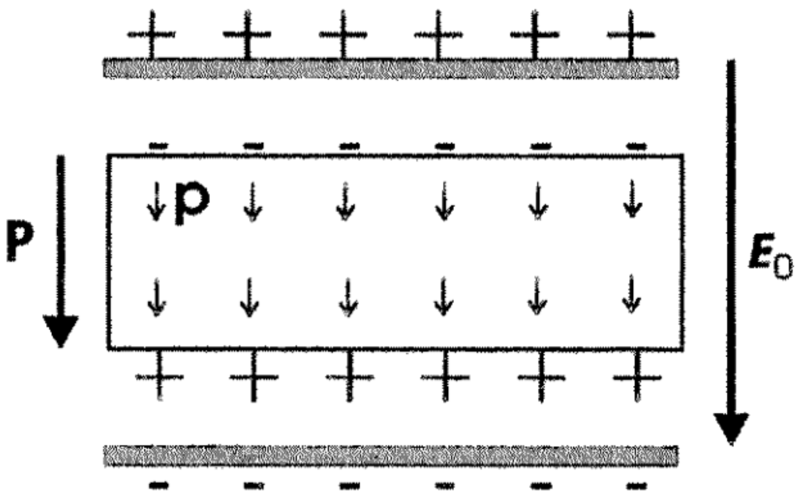
$$2nk = \varepsilon''$$

Polarisation Density

$$\mathbf{P} = \sum_j N_j \mathbf{p}_j = \sum_j N_j \alpha_j \mathbf{E}_{local}$$

$$\mathbf{p} = \mathbf{r} \cdot q$$

$$\mathbf{P} = \int \mathbf{r} q dV$$



E_{local} – local electric field

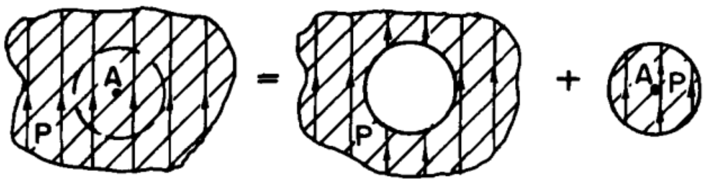
p_j – polarisation of one unit element of the matter (microscopic)

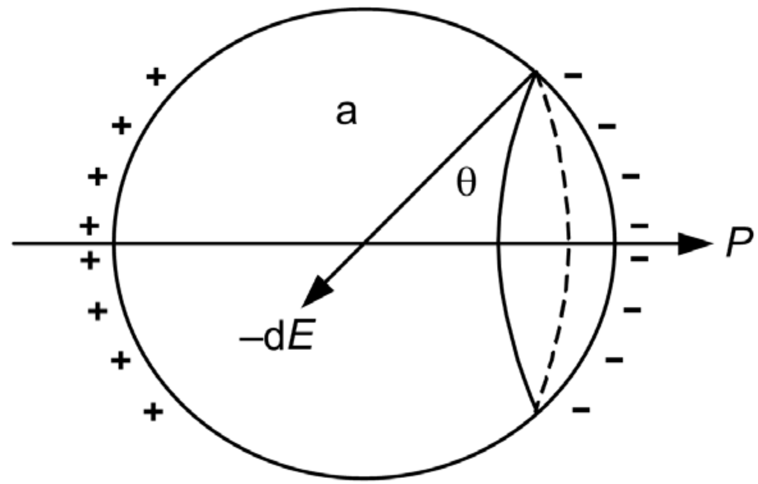
P – average (macroscopic) polarisation vector

N_j – concentration of the elements

α_j - polarisability of the element

Local electric field





for uniformly polarised medium (gas, liquid, glass, cubic crystal) one can show:

$$\mathbf{E}_{local} = \mathbf{E}_0 + \frac{\mathbf{P}}{3\varepsilon_0}$$

Lorentz formula

for lower symmetry crystals relation is more complex and ε will be a tensor

Relation between α and ε

$$\mathbf{E}_{local} = \mathbf{E}_0 + \frac{\mathbf{P}}{3\varepsilon_0}$$

$$\mathbf{P} = \sum_j N_j \alpha_j \mathbf{E}_{local}$$

$$\mathbf{P} = \varepsilon_0 \chi \mathbf{E}$$

⇒

$$\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{N\alpha}{3\varepsilon_0}$$ *Clausius-Mossotti relation*

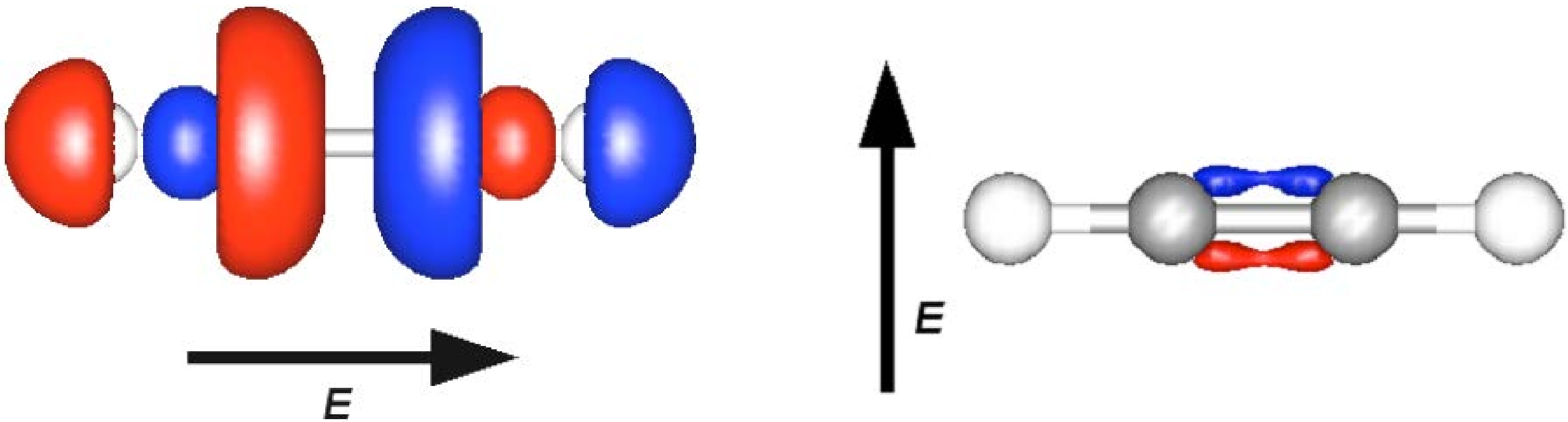
$$\alpha = 3\varepsilon_0 V_m \frac{\varepsilon - 1}{\varepsilon + 2} \qquad V_m = 1/N$$

Relation between polarisability of a unit element of the matter and dielectric constant

Dielectric response of the medium depends on the polarisability of microscopic elements (atoms, molecules) and their density

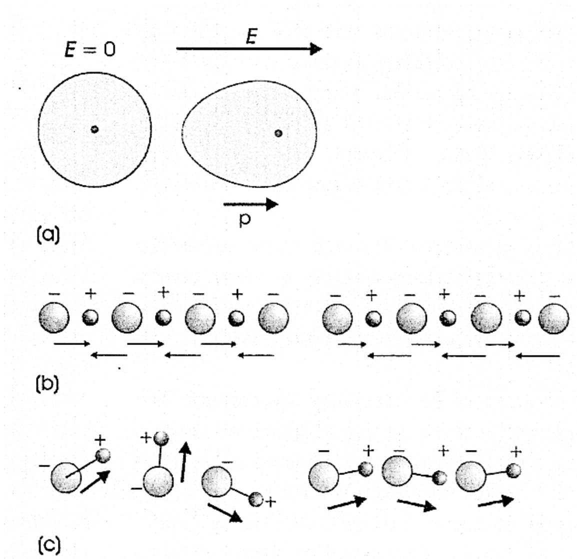
N α

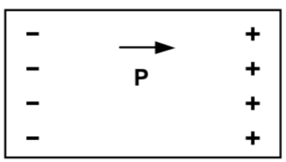
Example: C_2H_2



polarised C_2H_2 molecule

Polarisability α





- electronic polarisability
- ionic/atomic polarisability
- orientational polarisability
- space charge polarisability

$$\alpha = \alpha_{\text{charge}} + \alpha_{\text{orient.}} + \alpha_{\text{ionic}} + \alpha_{\text{electron.}}$$

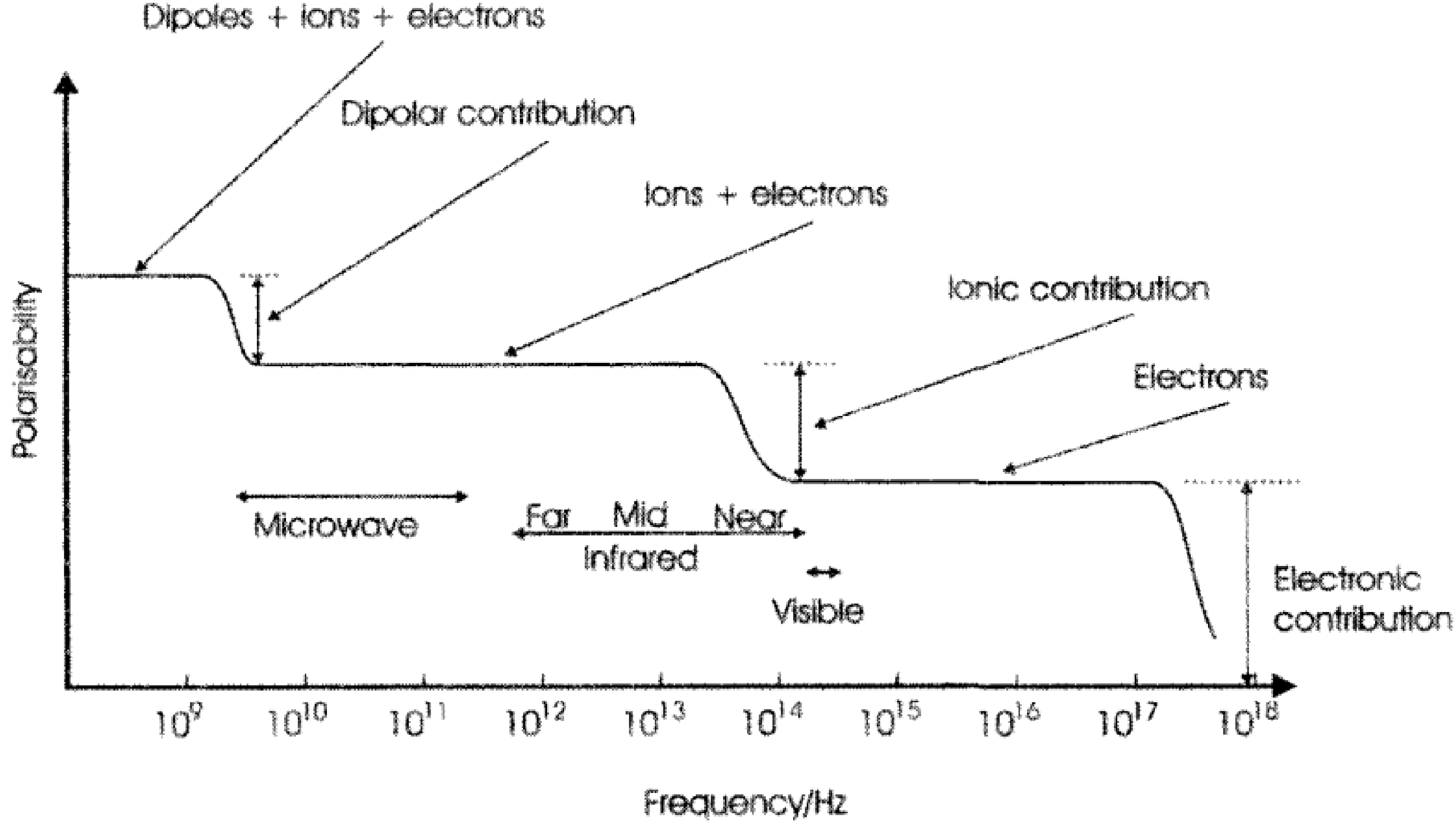
$$\mathbf{p} = \mathbf{r} \cdot q$$

$$\mathbf{P} = \int \mathbf{r} q dV$$

$$\mathbf{P} = \alpha \mathbf{E}_{local}$$

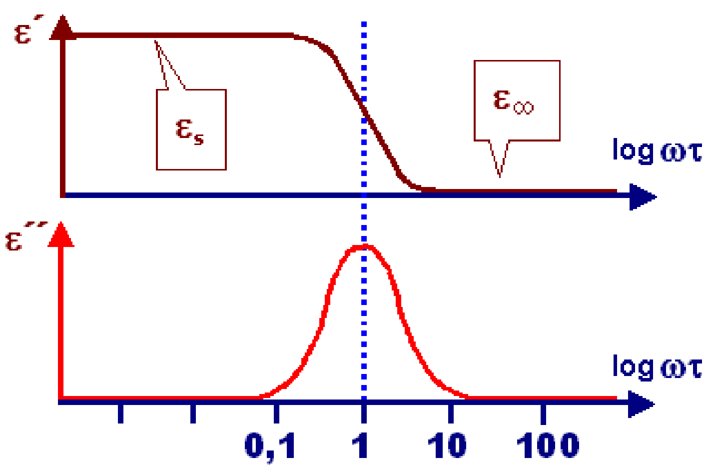
polarisability – ability to polarise under local (“true”) electric field (microscopic property)

Frequency dependence of α

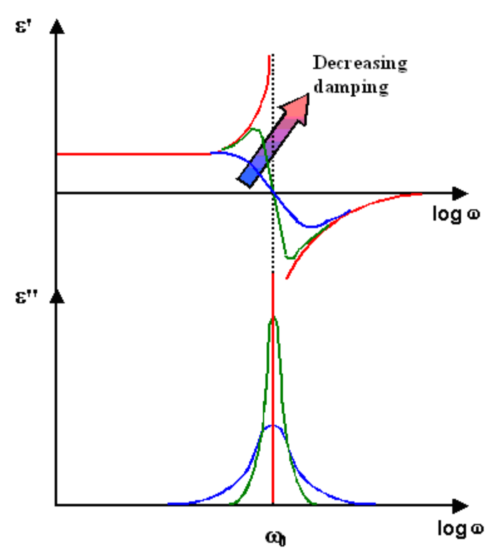


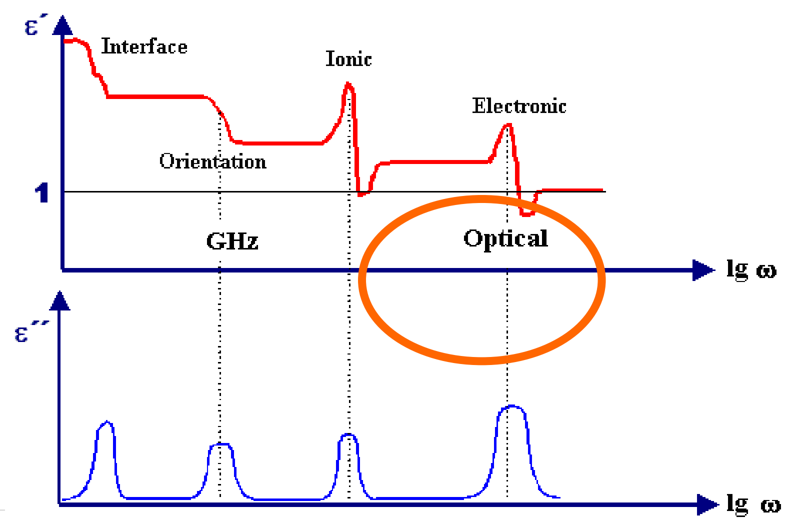
Resonances and Relaxations

Relaxation



Resonance





$$\varepsilon = \varepsilon' + i\varepsilon''$$

Frequency Dependance of Dielectric Function

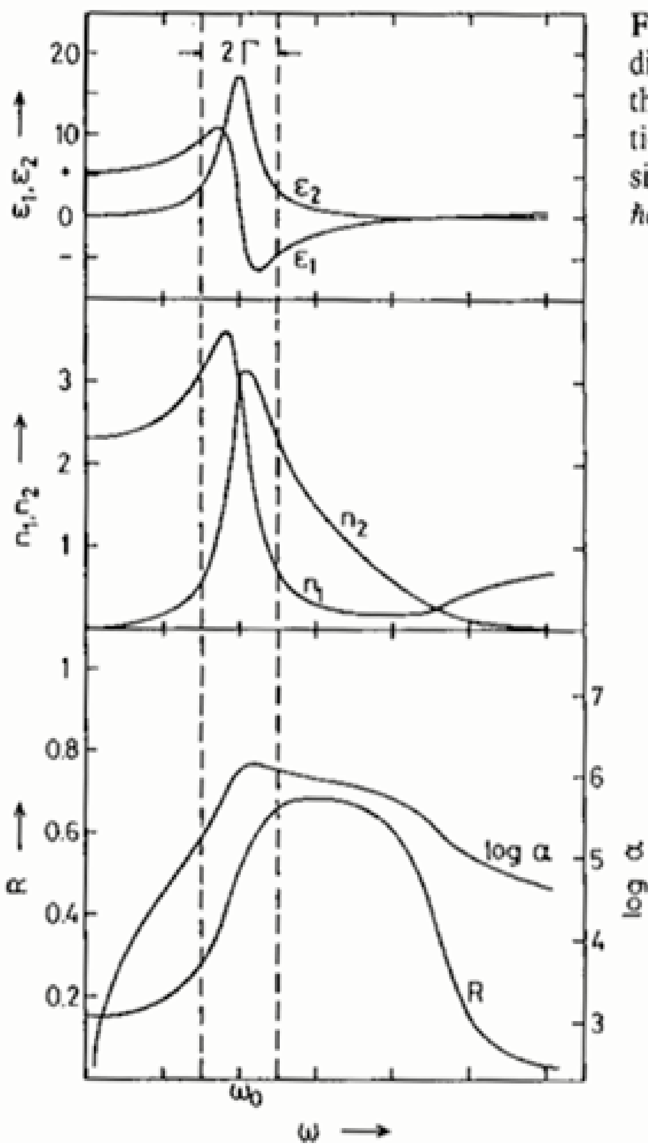


Fig.2.1. Frequency dependence of the dielectric function, the refractive index, the Fresnel reflectance and the absorption coefficient for a medium with a single resonance at ω_0 (calculated for $\hbar\omega_0 = 4\,\mathrm{eV}$, $\hbar\Gamma = 1\,\mathrm{eV}$, $N = 5\cdot 10^{22}\,\mathrm{cm}^{-3}$)

$$n_1 \equiv n$$
$$n_2 \equiv k_a$$

e

$$\varepsilon = 1 + \frac{N_e e_0^{\,2}}{m_e \varepsilon_0} f_{osc} \frac{\omega^2 - \omega_0^2 + i\Gamma\omega}{(\omega^2 - \omega_0^2)^2 - \Gamma^2\omega^2}$$

Electronic Polarisability of Atom

for optics in visible range only electronic polarisability important

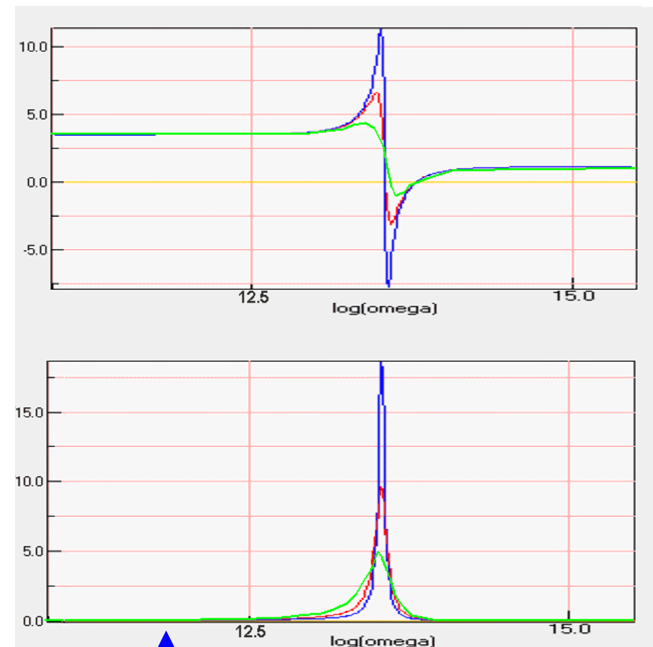
$$m\frac{d^2x}{dt^2}+\frac{m}{\tau}\frac{dx}{dt}+m\omega_0^2x=-q_eEe^{-i\omega t}$$

$$\Downarrow$$

$$x(t)=f(t)$$

$$\Downarrow$$

$$\alpha_{electron}=-\frac{q_ex}{E}=\frac{e^2}{m}\frac{1}{(\omega_0^2-\omega^2)-i\omega/\tau}$$



at low frequencies (ω→0)

$$\alpha_{\omega\to 0}=\frac{e^2}{m\omega_0^2}$$

What happens to the light?

- when interacting with the materials…

Light Properties

Or how to describe the electromagnetic wave ???
What parameters/properties do you know???

Let's start with exam !!!

Light Properties

Or how to describe the electromagnetic wave ???

Wave	Photon (Particle)

Collective (Beam) Properties

Light Properties

Or how to describe the electromagnetic wave ???

Wave	Photon (Particle)
Amplitude (ε)	Number of photons
Phase	Phase
Propagation Direction (k - vector)	Flight Direction
Wavelength / Frequency (λ / ν)	Energy [eV]
Polarisation (ψ, χ)	Polarisation

Collective (Beam) Properties
Power
Coherence length
Divergence of the beam
Monochromaticity
Polarization degree
Pulse length

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  - scattering

Recommended literature

- Optical properties of solids / Mark Fox. 2nd ed.. Oxford : Oxford University Press ; 2010

Absorption & Luminescence

Absorption

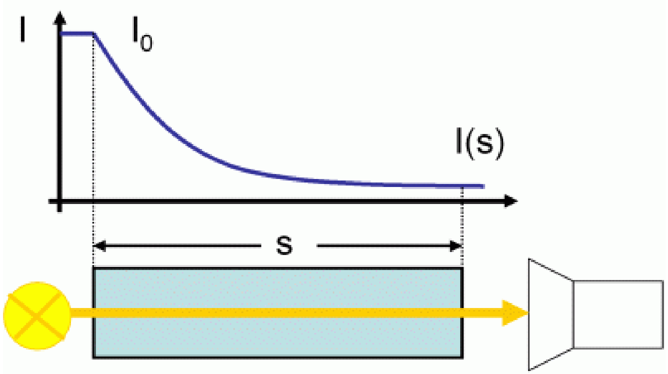
for linear absorption lost intensity in a thin layer is proportional to incident intensity

$$\frac{dI(z)}{dz} = -\alpha \cdot I(z)$$

which results in a Beer-Lambert law

$$I = I_0 \cdot e^{-\alpha \cdot z} = I_0 \cdot e^{-z/l_\alpha}$$

α - absorption coefficient
$l_\alpha = 1/\alpha$ - penetration depth
(characteristic absorption length)



Measures of Absorption

absorption coefficient and extinction coefficient

$$\alpha = 4\pi \frac{k}{\lambda}$$

optical density – a logarithmic unit:

$$O.D. = -\log_{10} \frac{I(l)}{I_0} = \frac{\alpha \cdot l}{\log_e 10}$$

complex refractive index and complex dielectric constant:

$$\tilde{n}^2 = (n+ik)^2 = \varepsilon' + i\varepsilon'' = \tilde{\varepsilon}$$

$$n^2 - k^2 = \varepsilon'$$

$$2nk = \varepsilon''$$

$$n = \sqrt{\frac{\varepsilon' + \sqrt{\varepsilon'^2 + \varepsilon''^2}}{2}}, k = \sqrt{\frac{-\varepsilon' + \sqrt{\varepsilon'^2 + \varepsilon''^2}}{2}}$$

Absorption

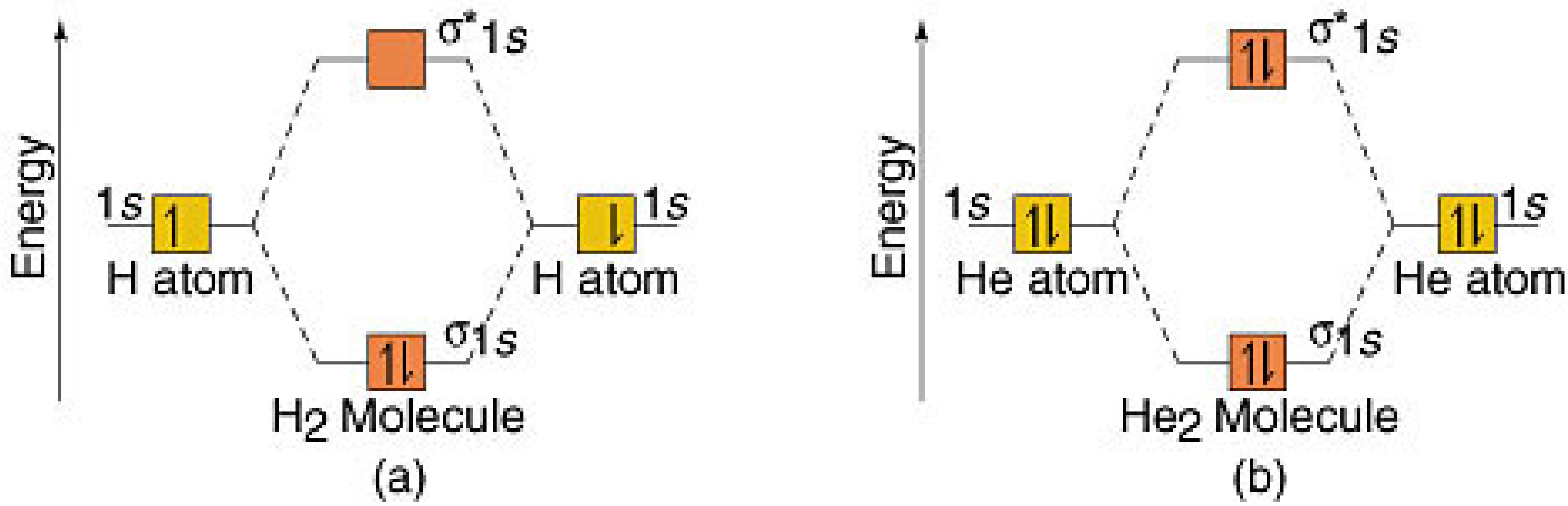
$$I = I_0 \cdot e^{-\alpha \cdot z} = I_0 \cdot e^{-z/l_\alpha}$$

macroscopic ("black box") description of absorption

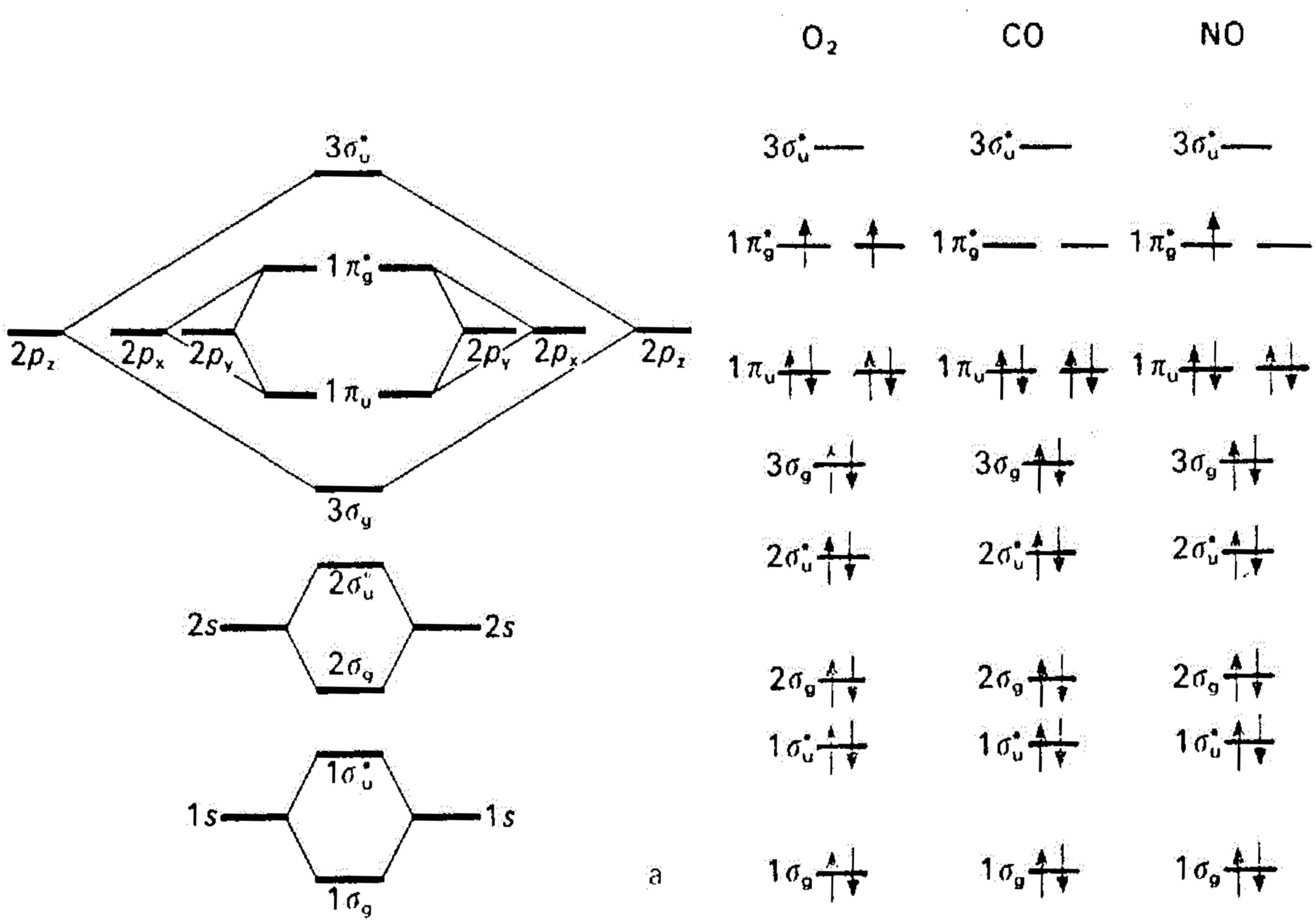
Types of Absorption /Excitation

Absorption	Wavelength (nm)
electronic molecular(σ, π)	150 – 1-2μm
electronic interband	150 – 1-2μm
vibrational	900 – 10'000
rotaional	> 10'000

Energy Levels



Light absorption in a Gaseous Molecule

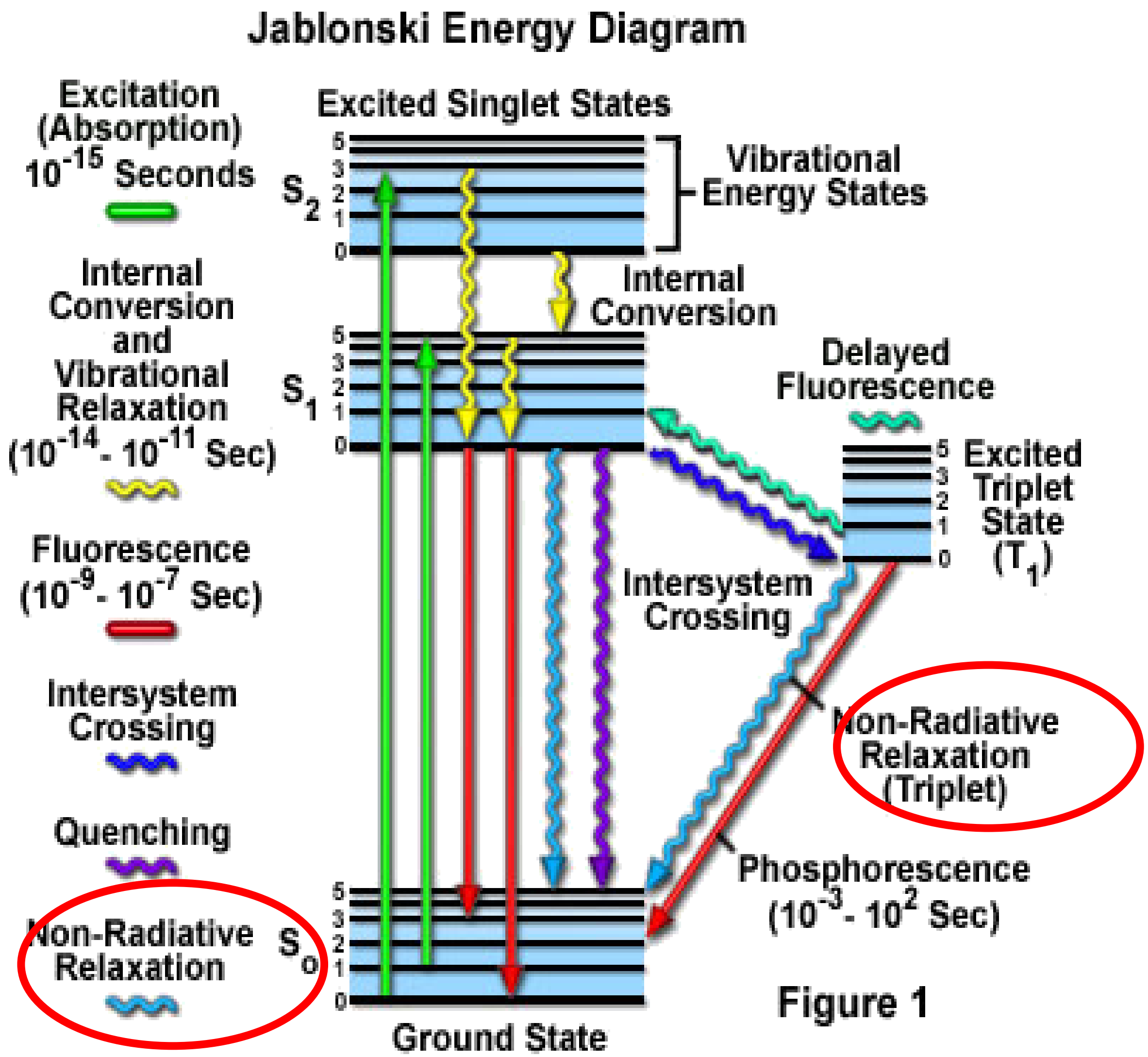


Absorption & Fluorescence

Absorption & fluorescence in a single molecule, ion, atom

!

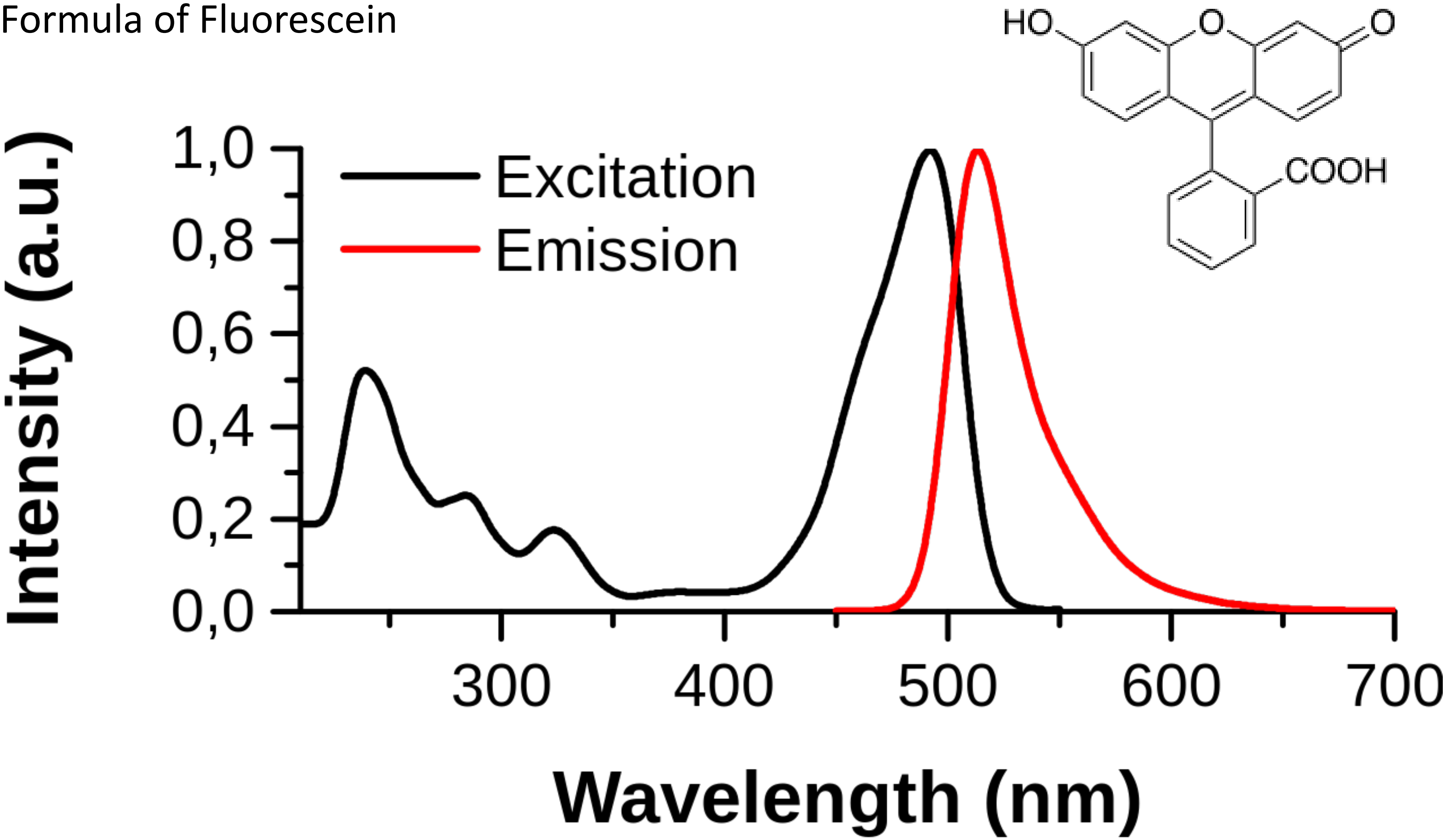
non-radiative transition → transformation into heat



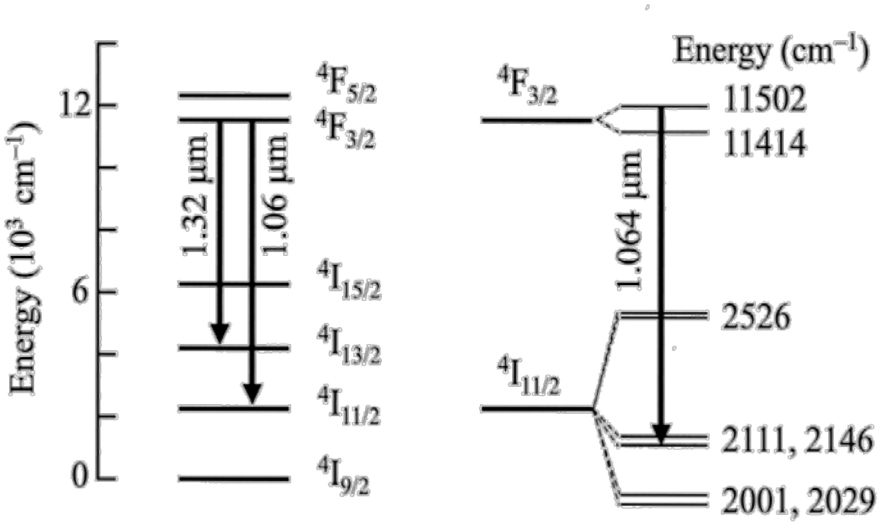
http://micro.magnet.fsu.edu/primer/techniques/fluorescence/fluorescenceintro.html

Fluorescein

Formula of Fluorescein

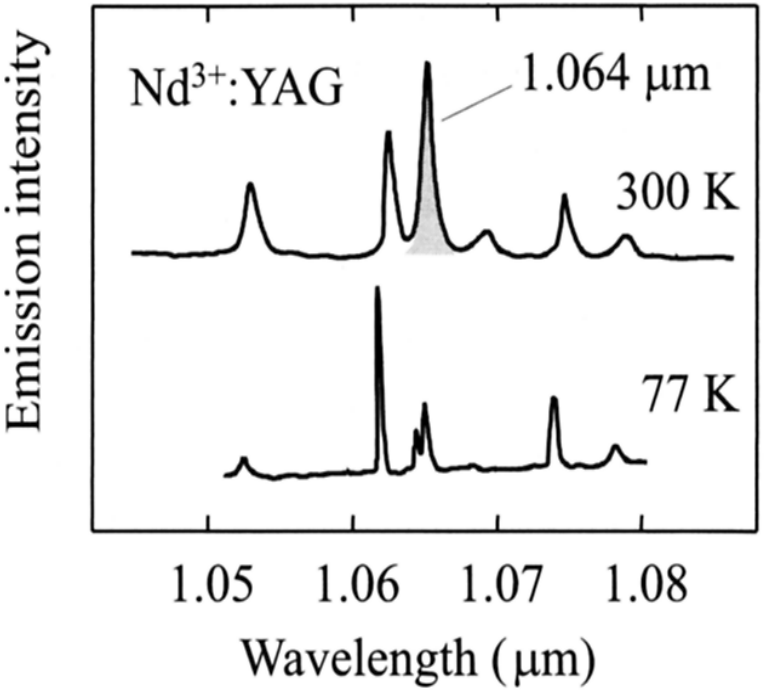


Absorption/Luminescence Centers in Solids

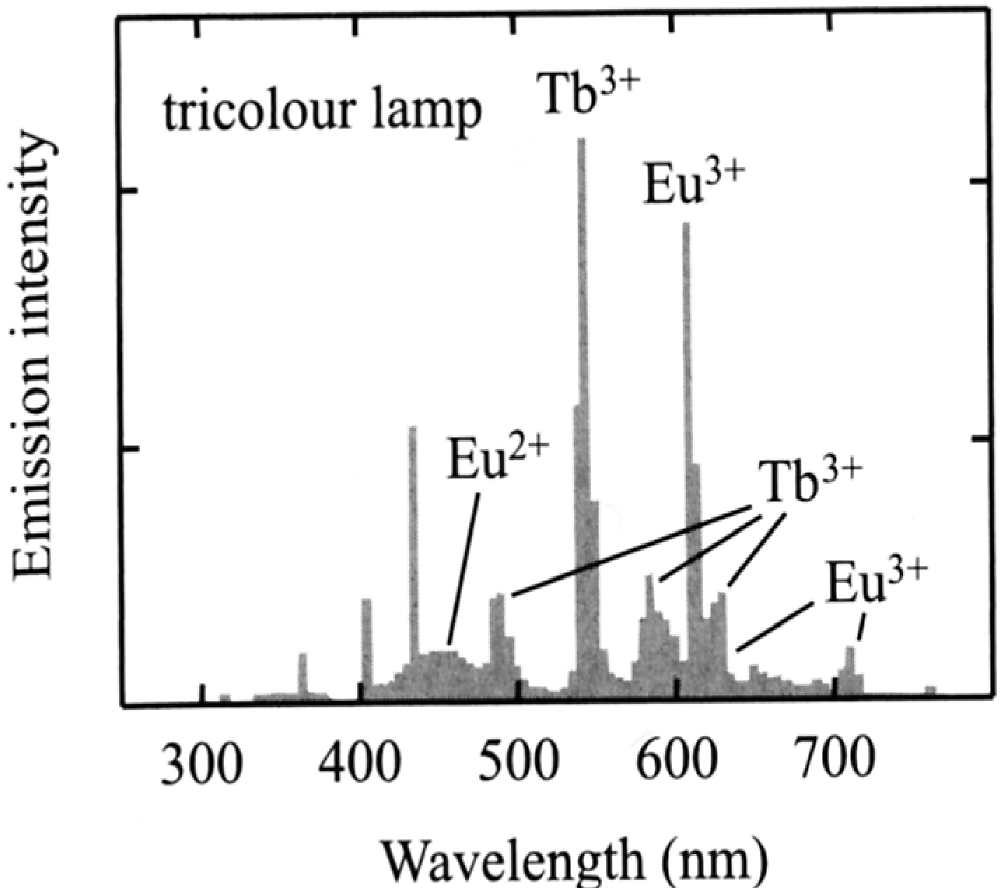


Energy Level Diagram of Nd^{3+}-ion in YAG crystal

Emission spectrum of Nd^{3+}-ion in YAG crystal



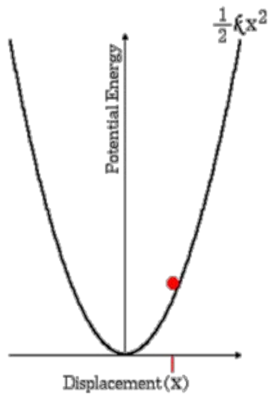
Emission of Luminescent Lamp

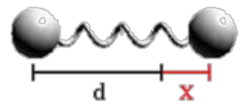


emission of luminescent lamp is composed of narrow line emission of a mixture of phosphors

Vibronic Transitions and Infrared Spectroscopy

Vibration of a di-atomic molecule

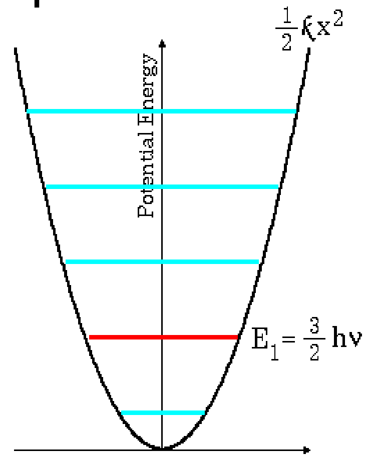




The **vibration frequency** ν is related to the force constant k and mass m by the equation:

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

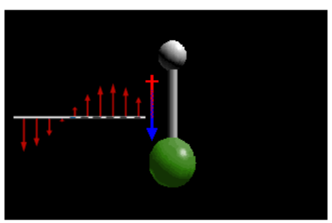
quantized harmonic oscillator



In the quantum mechanical model, a molecule in may only absorb (or emit) light of an energy equal to the spacing between two levels. Furthermore, for a harmonic oscillator these transitions can only occur from one level to the next higher (or lower) level, i.e. **Δn = ±1**. This is called the **selection rule**. Press the buttons below to see absorbtion and emission:

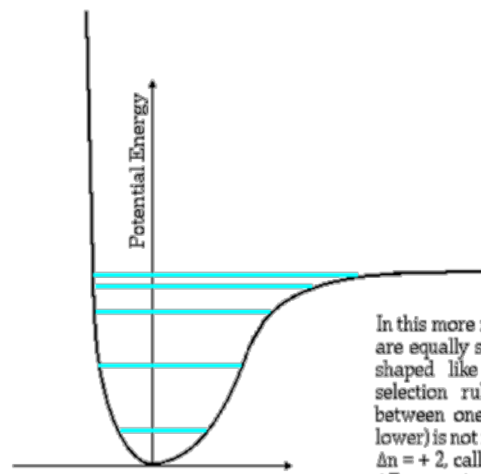
Absorb Emit

Selection rules for oscillation to be visible in IR spectroscopy: the dipole moment of a molecule have to change when undergoing the transition



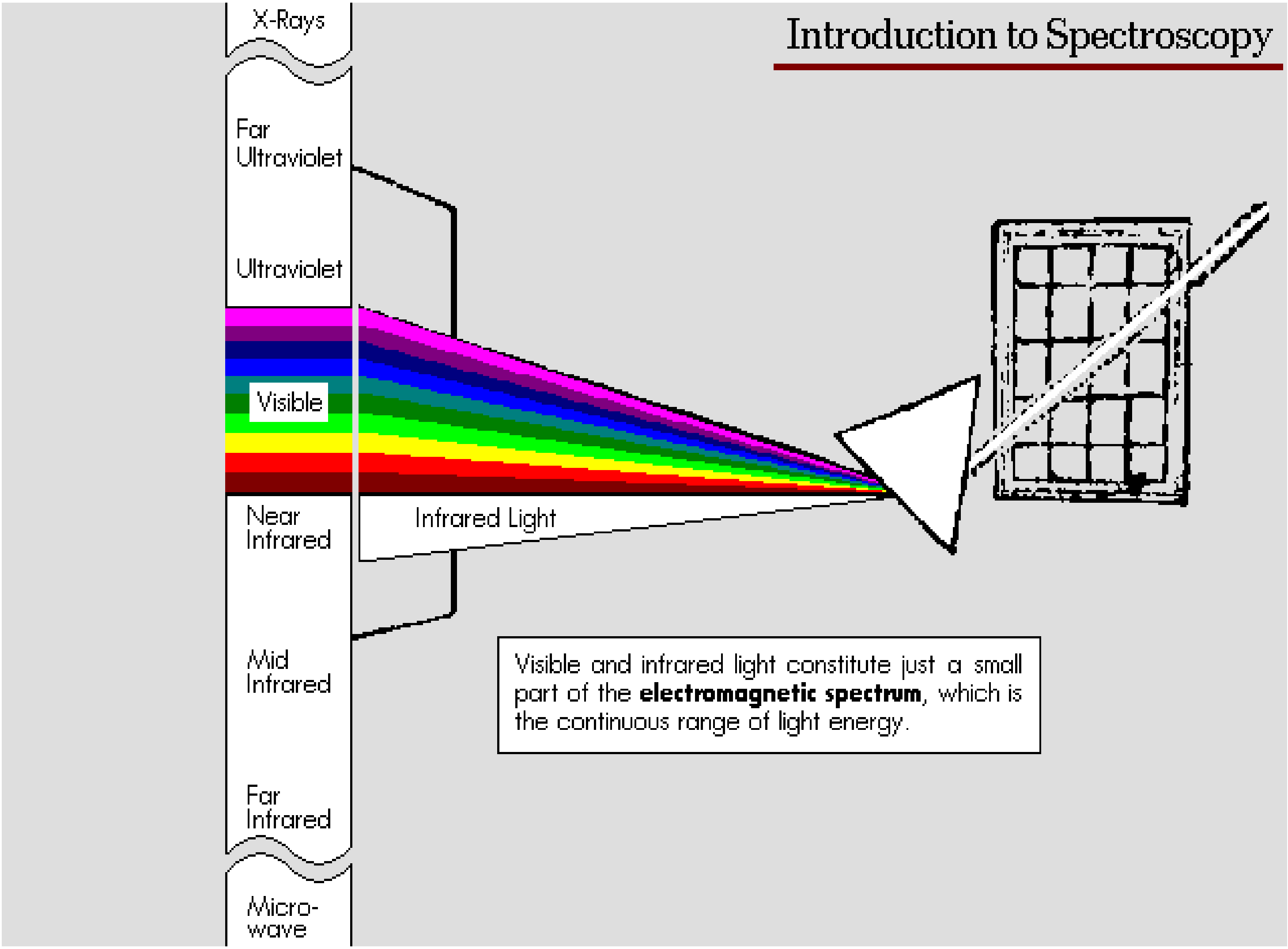
However, HCl does have a dipole change as it stretches. When this dipole aligns with the electric field of a beam of light, the light is absorbed (so long as the frequency is correct). The intensity of the absorption is related to the magnitude of the dipole change.

quantized unharmonic oscillator



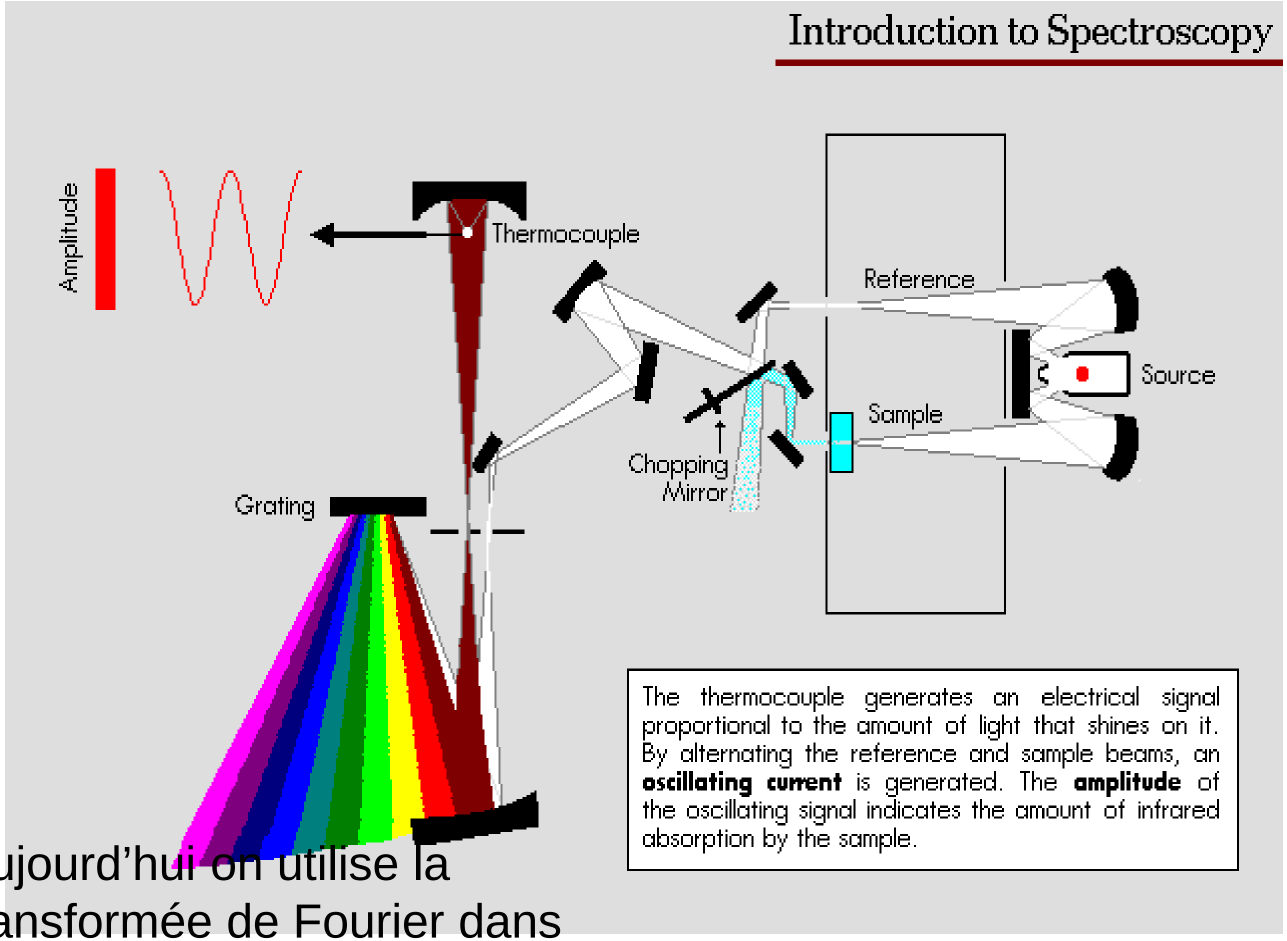
In this more realistic model, the energy levels are equally spaced only in the region that is shaped like the harmonic potential. The selection rule, which allowed transitions between one level and the next higher (or lower) is not rigorously true. A transition with Δn = + 2, called an **overtone**, corresponds to ΔE approximately 2 hν.

Infrared Spectroscopy



Schematic of Infrared Spectroscopy

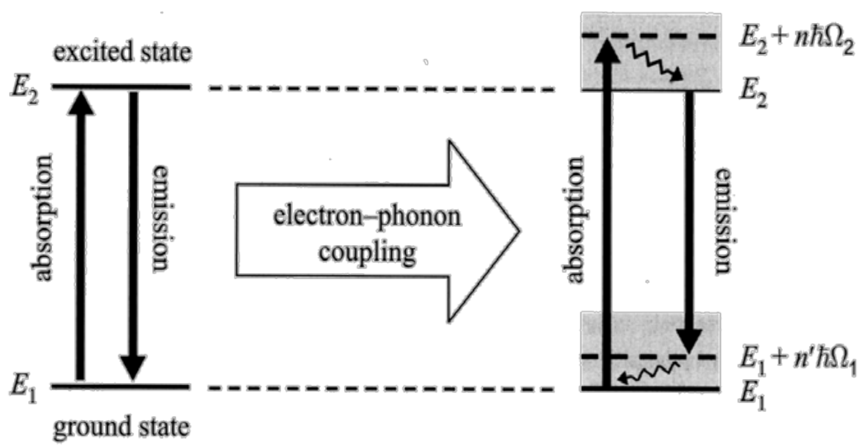
En pivotant le réseau (en anglais *grating*, prisme réflectif), la longueur d'onde arrivant sur l'échantillon est scannée. L'absorption dans l'échantillon varie avec la longueur d'onde.
On obtient sur le détecteur un spectre IR.



Aujourd'hui on utilise la transformée de Fourier dans les machines, pour augmenter la vitesse et la precision.

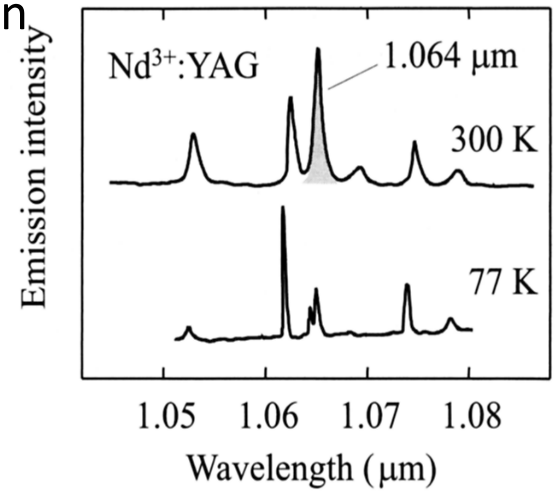
Electronic and Vibronic Trasitions

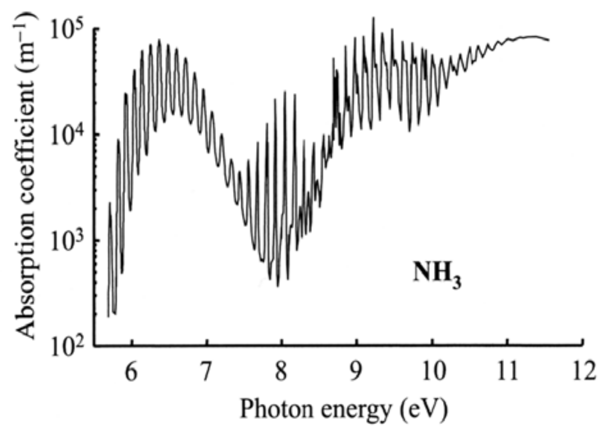
electronic and vibronic transition often interact – broadening of the transitions (emission / absorption peaks)



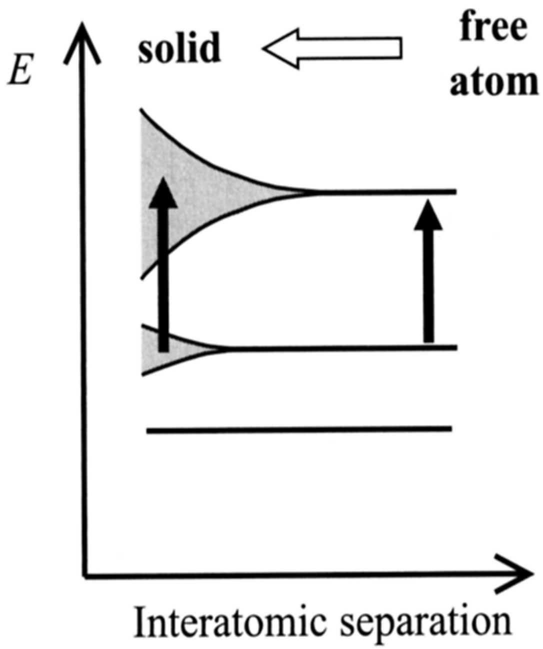
(a) Isolated atom

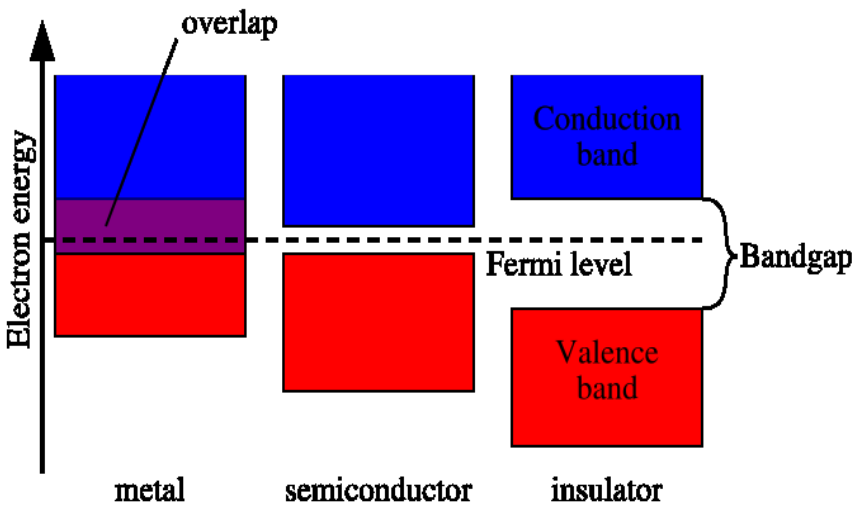
(b) Atom doped into a vibronic solid





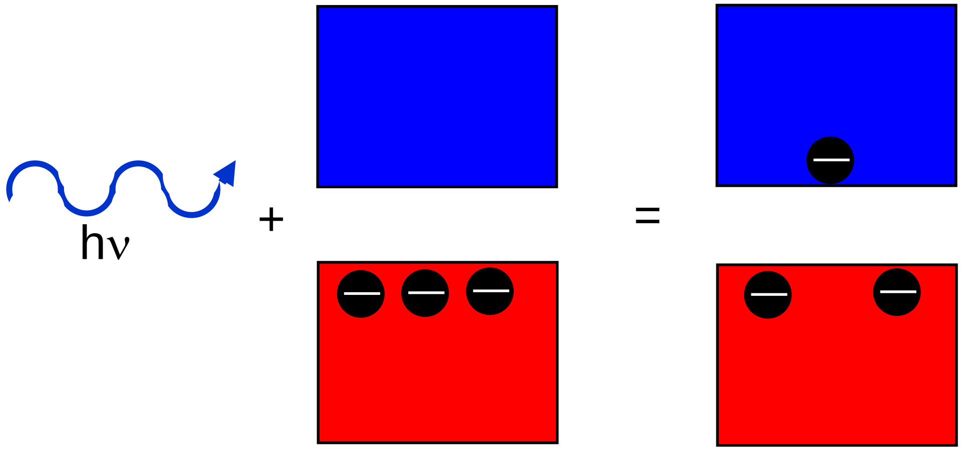
Band Structure of Solids





http://en.wikipedia.org/wiki/Band_structure

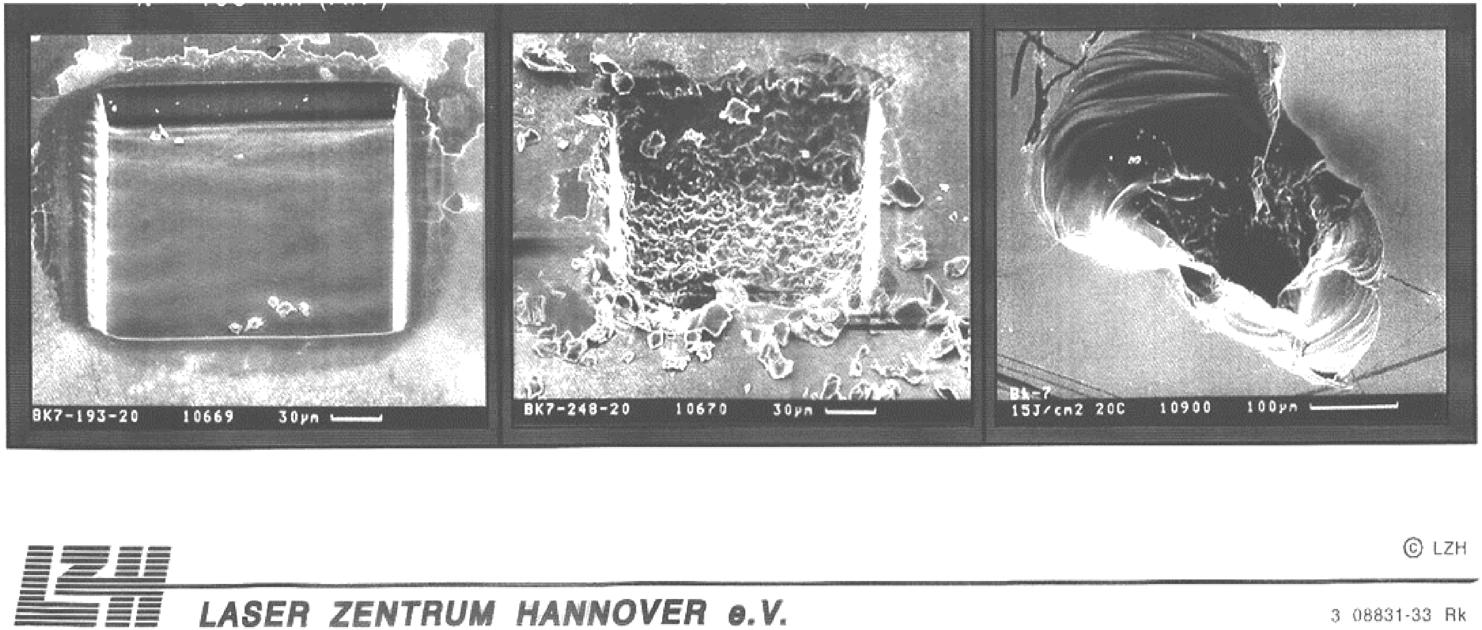
Band Gap Absorption



Only photons with energy larger than the band gap of material are absorbed
Inter band absorption grates free charge carriers – new centers of absorption

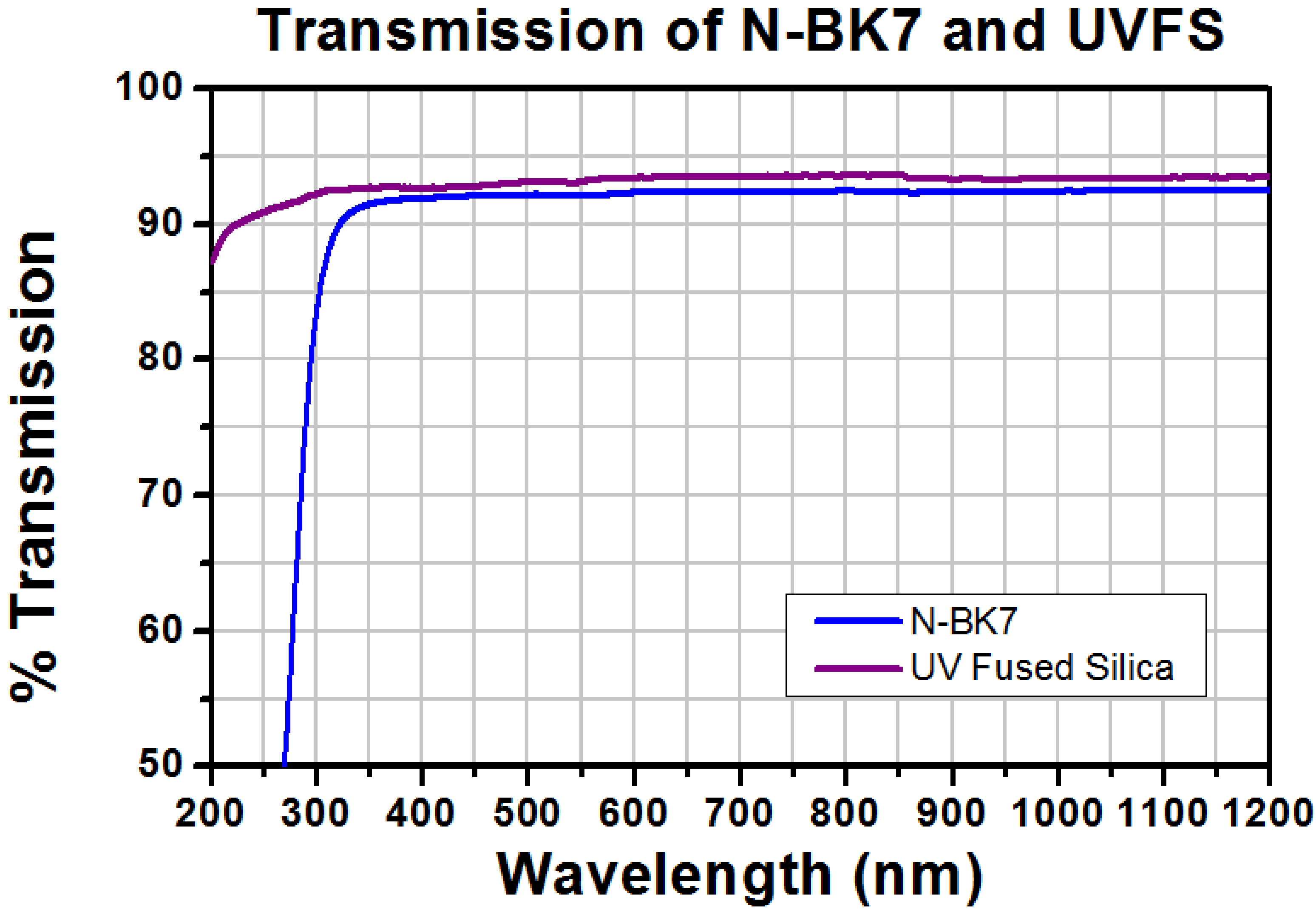
Comparison Excimer Ablation of BK7 Glass

λ = 193 nm (ArF) λ = 248 nm (KrF) λ = 308 nm (XeCl)



Where is the absorption edge (band gap) of BK7 glass?

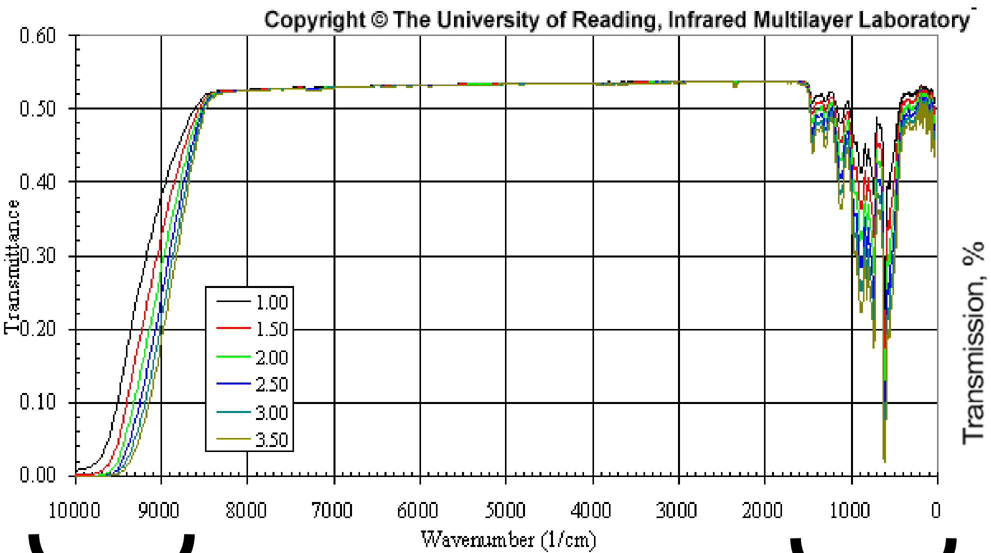
Material: BK7 glass
Energy density: 20J/cm^2
Pulse frequency: 50 Hz
Number of pulses: 50



Transmission of N-BK7 and UVFS
% Transmission
100
90
80
70
60
50
200
300
400
500
600
700
800
900
1000
1100
1200
Wavelength (nm)
N-BK7
UV Fused Silica

Transmission Spectra (Silicon)

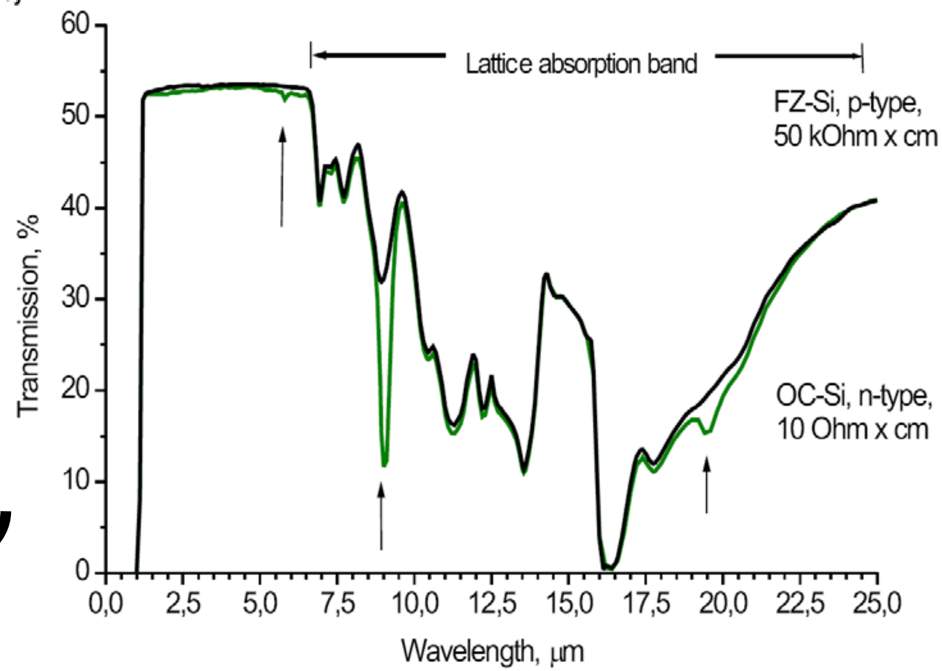
Calculated transmission profiles of hyperpure Fz Silicon (Si) at 293K for substrate thicknesses between 1.0 and 3.5mm



Absorption due to electronic band to band transitions (band gap)

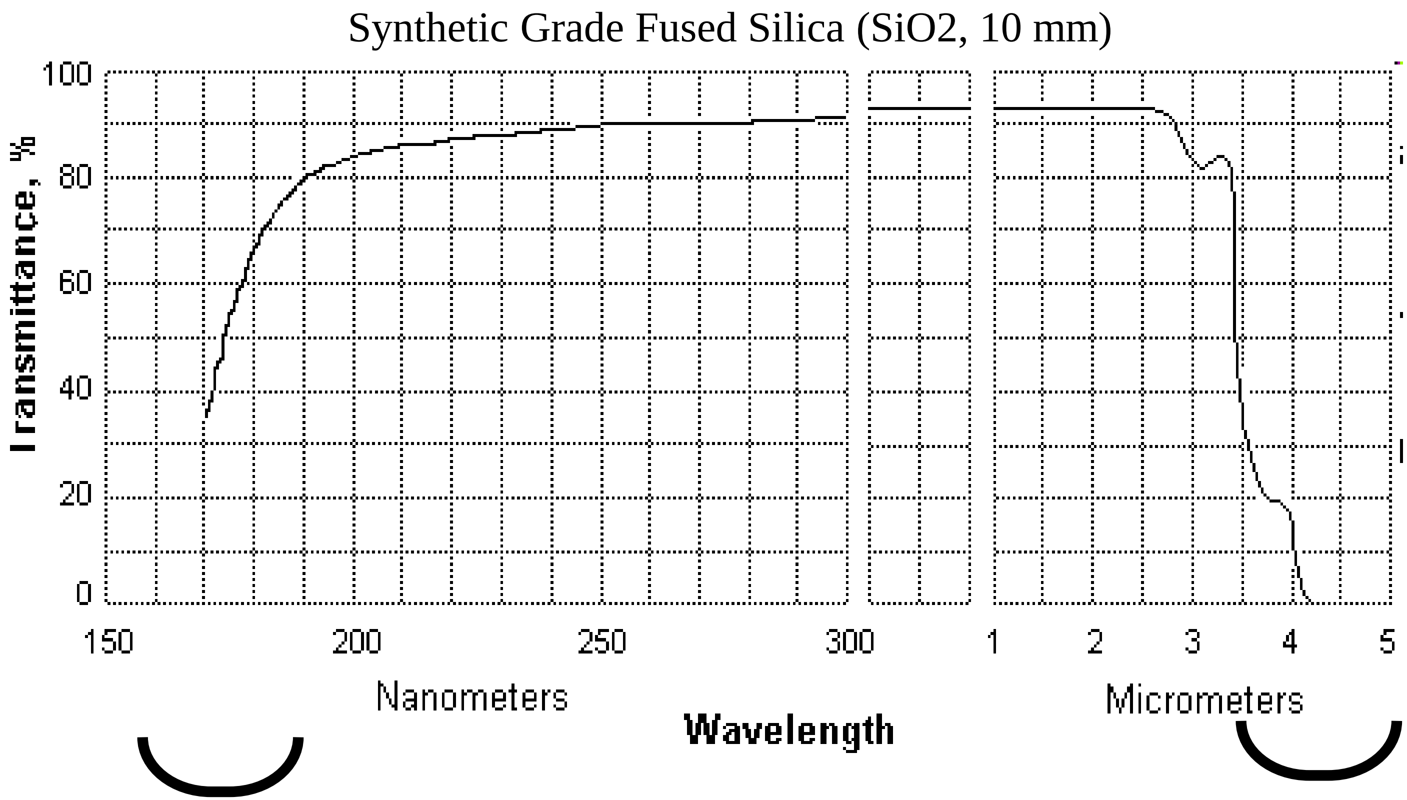
Absorption due to excitation of lattice vibrations (phonons)

Silicon (5 mm) transmission. (pure and oxygen contaminated) Arrows point to the oxygen absorption peaks.



Tydex J.S.Co
http://www.tydex.ru

Transmission Spectra (Fused Silica)



Absorption due to electronic band to band transitions (band gap)

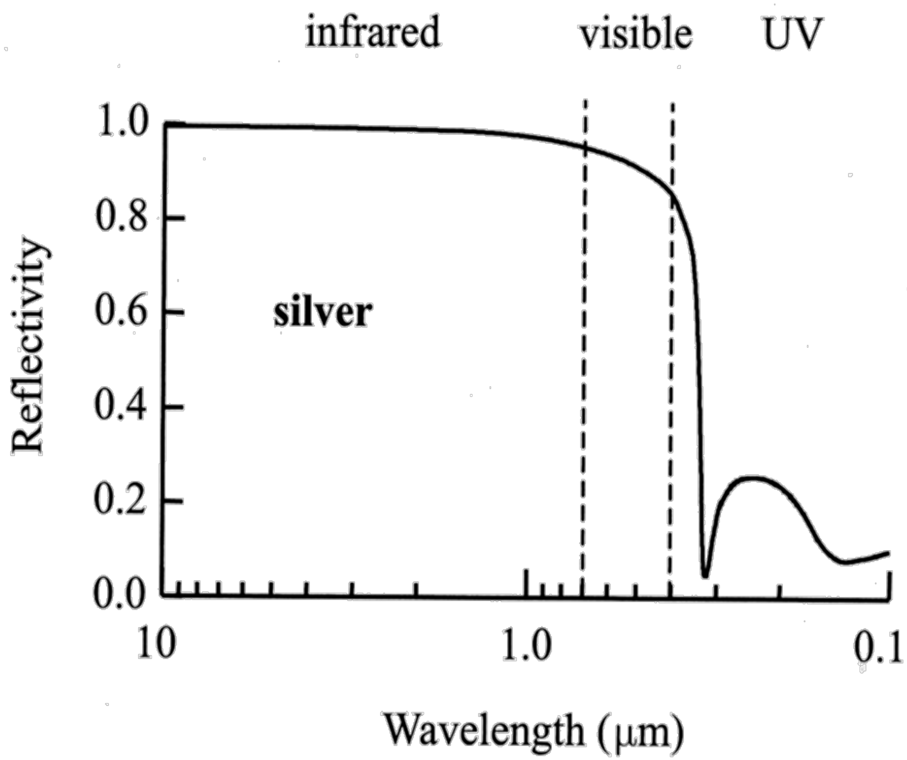
Absorption due to excitation of lattice vibrations (phonons)

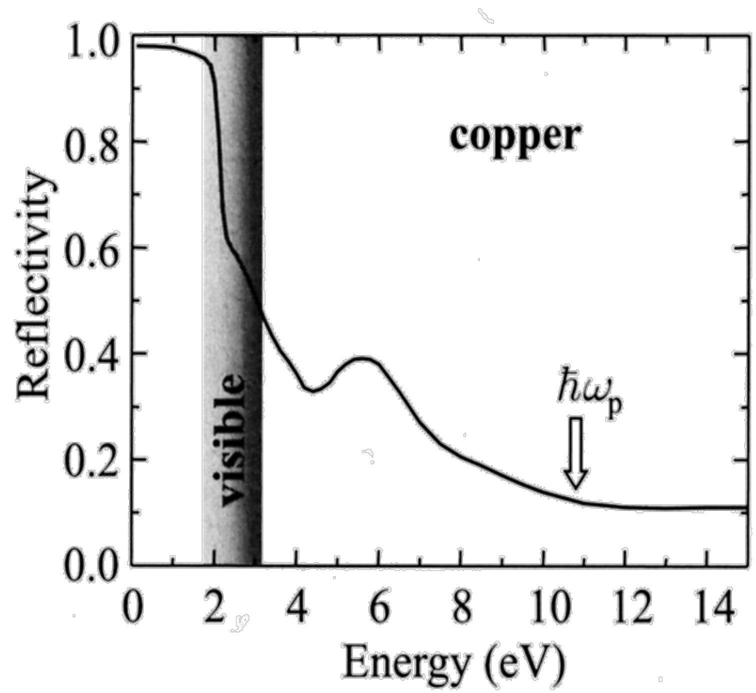
Almaz Optics Inc.
http://www.almazoptics.com

Refractive index

Optical properties of Metals

Metals are shiny and typically not transparent



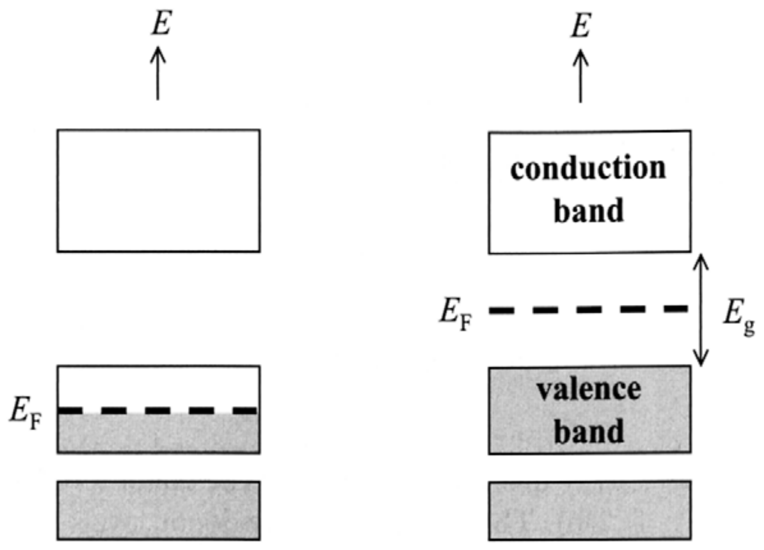


What would be the best laser for metal processing?

TRUMPF CO_2 Laser

Why in reality other lasers are used?

Band Structure of Metals



(a) Metal

(b) Semiconductor or insulator

in metal electrons are not bounded ⇒ free-electron plasma

Electrons in Metal

in metal electrons are not bounded $\Rightarrow$ free-electron plasma

$$m_0 \frac{d^2x}{dx^2} + m_0\gamma\frac{dx}{dt} + \cancel{m_0\omega^2 x} = -eE(t) = -eE_0 e^{-i\omega t}$$

acceleration damping E-field driving force

"spring" (returning force) term is not present

solution:

$$x = \frac{e}{m_0(\omega^2 + i\omega\gamma)}E(t)$$

$$P = -N_e \cdot e \cdot x = \varepsilon_0(\varepsilon - 1)E$$

dielectric constant for free-electron plasma (in first approximation for metals):

$$\varepsilon(\omega) = 1 - \frac{N_e e^2}{\varepsilon_0 m_0}\frac{1}{(\omega^2 + i\gamma\omega)} = 1 - \frac{\omega_p}{(\omega^2 + i\gamma\omega)}$$

$$\omega_p = \sqrt{\frac{N_e e^2}{\varepsilon_0 m_0}}$$

plasma frequency

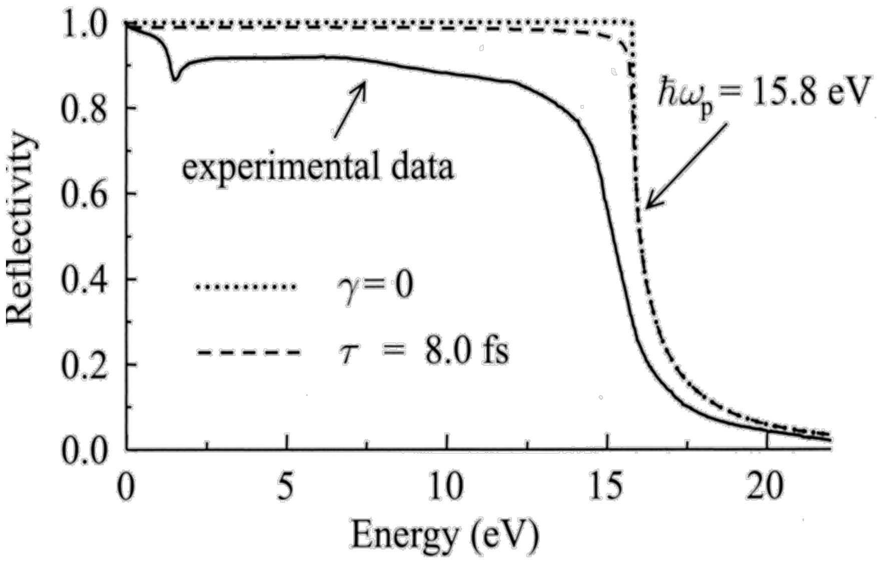
Theoretical Plasma Frequencies

Table 7.1 Free electron density and plasma properties of some metals. The figures are for room temperature unless stated otherwise. The electron densities are based on data taken from Wyckoff (1963). The plasma frequency ω_p is calculated from eqn 7.6, and λ_p is the wavelength corresponding to this frequency.

Metal	Valency	N (10^{28} m^{-3})	$\omega_p/2\pi$ (10^{15} Hz)	λ_p (nm)
Li (77 K)	1	4.70	1.95	154
Na (5 K)	1	2.65	1.46	205
K (5 K)	1	1.40	1.06	282
Rb (5 K)	1	1.15	0.96	312
Cs (5 K)	1	0.91	0.86	350
Cu	1	8.47	2.61	115
Ag	1	5.86	2.17	138
Au	1	5.90	2.18	138
Be	2	24.7	4.46	67
Mg	2	8.61	2.63	114
Ca	2	4.61	1.93	156
Al	3	18.1	3.82	79

$$\varepsilon(\omega) = 1 - \frac{\omega_p}{\left(\omega^2 + i\gamma\omega\right)}$$

reflectivity of free-electron gas and real metal (Al)



$$\gamma = 1/\tau$$

γ – damping coefficient

τ – momentum scattering time

Dielectric Constant and Conductivity

$$\varepsilon(\omega) = 1 - \frac{\omega_p}{\omega^2 + i\gamma\omega}$$

$$\varepsilon(\omega) = 1 + \frac{i\sigma(\omega)}{\varepsilon_0\omega},$$

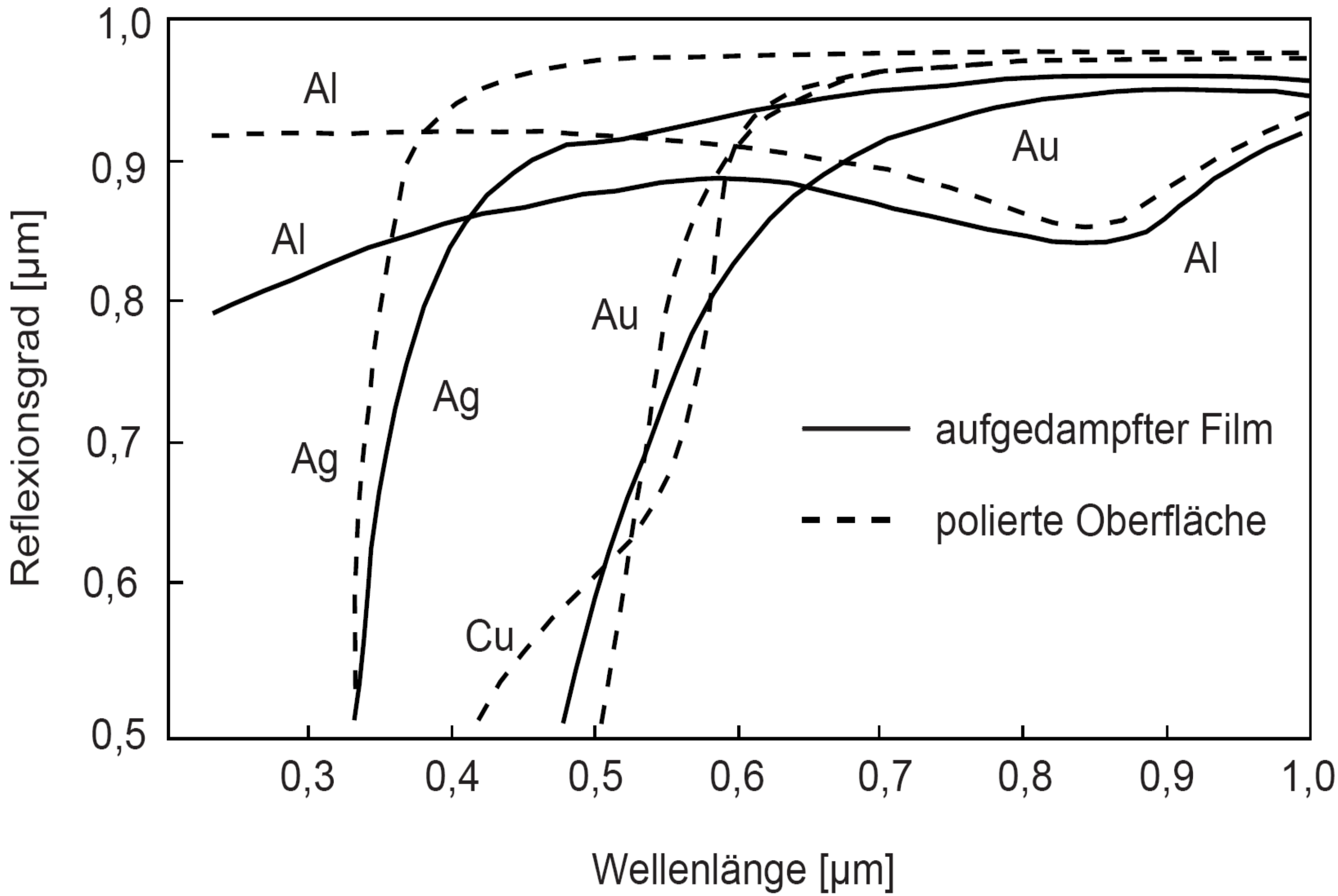
$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau} - \text{ AC conductivity}$$

$$\sigma_0 = \frac{N_e e^2 \tau}{m_0} - \text{ DC conductivity}$$

$$\sigma_0 = \omega_p^2 \varepsilon_0 \tau$$

conductivity and plasma frequency of a metal are directly linked (link between optical and electrical properties)

Reflection of Al, Ag & Au



Reflection coefficients and penetration depth for some Materials

maté-riaux	l_α (250 nm)	R (250 nm)	l_α (500 nm)	R (500 nm)	l_α (1060 nm)	R (1060 nm)	l_α (10.6 µm)	R (10.6 µm)
KCl	> 1 cm	0.05	> 1 cm	0.04	> 1 cm	0.04	> 1 cm	0.03
SiO_2	> 1 cm	0.06	> 1 cm	0.04	> 1 cm	0.04	40 µm	0.2
Ge	7 nm	0.42	15 nm	0.49	200 µm	0.38	1 mm	0.36
Si	6 nm	0.61	500 nm	0.36	200 µm	0.33	1 mm	0.3
Ag	20 nm	0.30	14 nm	0.98	12 nm	0.99	12 nm	0.99
Al	8 nm	0.92	7 nm	0.92	10 nm	0.94	12 nm	0.98
Au	18 nm	0.33	22 nm	0.48	13 nm	0.98	14 nm	0.98
W	7 nm	0.51	13 nm	0.49	23 nm	0.58	20 nm	0.98

for metals for frequencies between:
$1/\tau_e < \omega < \omega_p$ (optical, vis.)

$$\alpha \approx \frac{2\omega_p}{c}$$

$$l_\alpha \approx \frac{c}{2\omega_p}$$

Table 1: Energies de gap pour les examples

Matériaux	Gap (eV)	λ (nm)
Si	1.1	1130
SiO_2	6.9	180

Ablation of Biotissue - Difference in Absorption

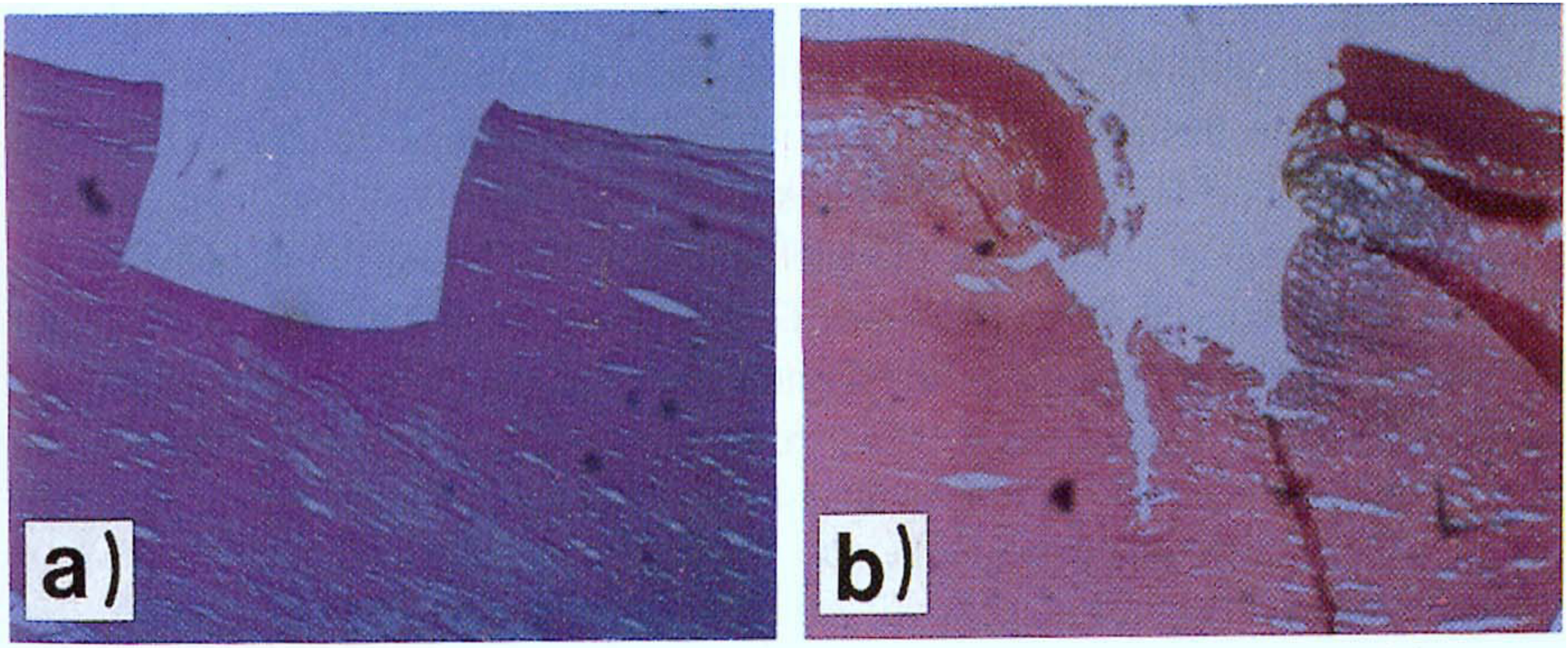
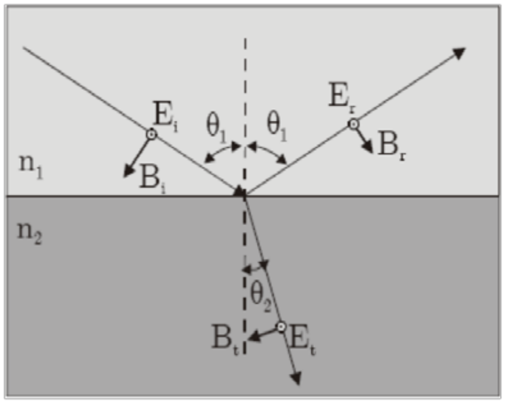


Fig. 12.1.5a,b. Cross section of luminal side of an aortic wall. (**a**) Trench (0.35 mm) produced by ArF-laser radiation ($\phi \approx 0.25$ J/cm^2, $\tau_l \approx 14$ ns). (**b**) Crater (0.4 mm) produced by 532 nm Nd:YAG laser radiation ($\phi \approx 1.0$ J/cm^2, $\tau_l \approx 5$ ns). The absorption coefficients of the material at the two wavelengths differ by about a factor of 10^3 [*Srinivasan* 1986]

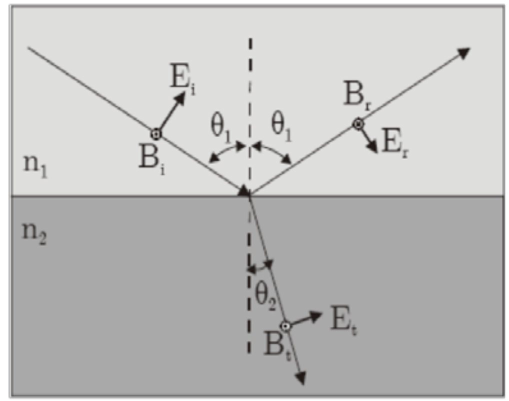
Reflection and Refraction

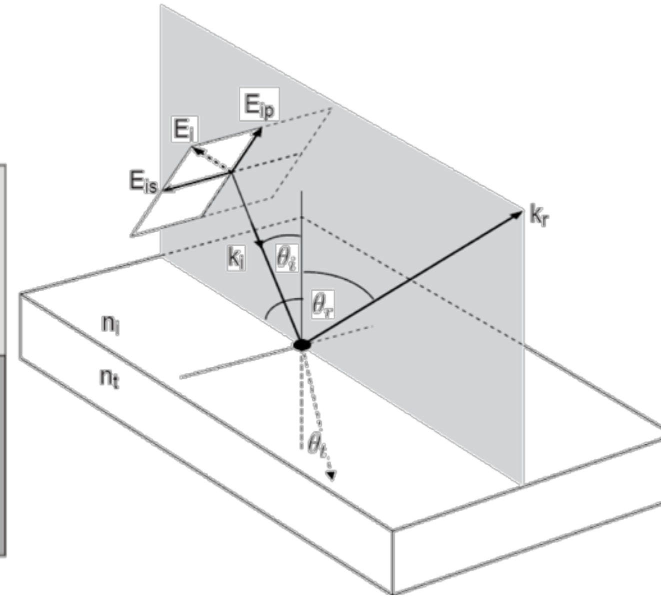
Reflection - Fresnel Equations

σ-polarization (TE)
(Transversal Electric field)



π-polarization (TM)
(Transversal Magnetic field)





$$r^{\sigma}=\frac{n_1\cos(\theta_1)-n_2\cos(\theta_2)}{n_1\cos(\theta_1)+n_2\cos(\theta_2)} \quad (B.1)$$

$$r^{\pi}=\frac{n_1\cos(\theta_2)-n_2\cos(\theta_1)}{n_1\cos(\theta_2)+n_2\cos(\theta_1)} \quad (B.3)$$

$$t^{\sigma}=\frac{2n_1\cos(\theta_1)}{n_1\cos(\theta_1)+n_2\cos(\theta_2)} \quad (B.2)$$

$$t^{\pi}=\frac{2n_1\cos(\theta_1)}{n_1\cos(\theta_2)+n_2\cos(\theta_1)} \quad (B.4)$$

Field amplitude coefficients

Fresnel Equations

Light intensity (Power) coefficients:

$$R^{\sigma} = \left(r^{\sigma}\right)^2$$

$$R^{\pi} = \left(r^{\pi}\right)^2$$

In case of normal incidence:

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

Do not forget to use complex refractive index, if absorption is present!

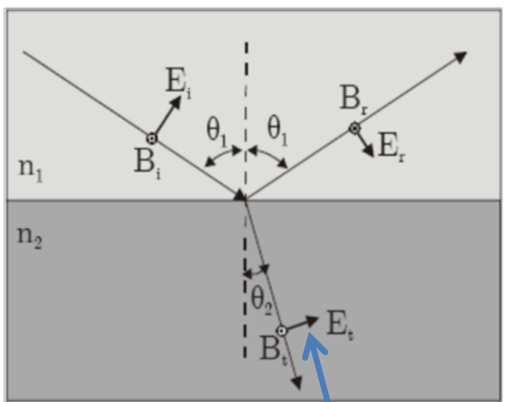
$$R = \left(\frac{\tilde{n}_1 - \tilde{n}_2}{\tilde{n}_1 + \tilde{n}_2}\right)^2$$ normal incidence

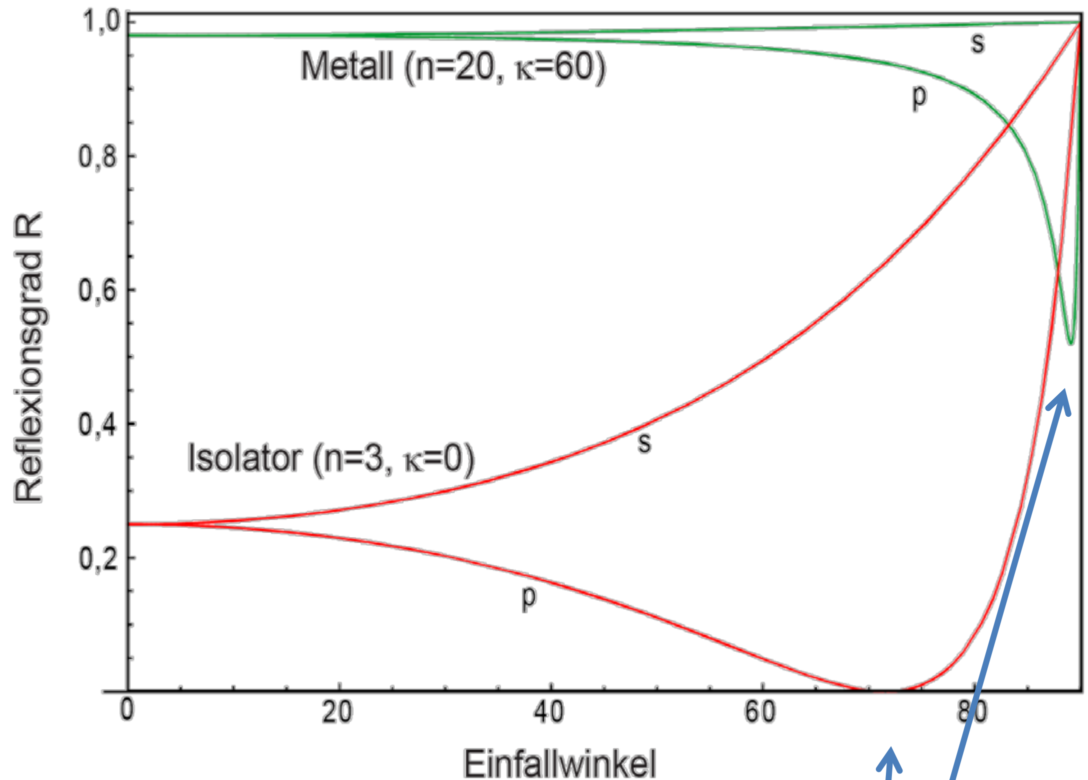
$$R^{\sigma} = \left(\frac{\tilde{n}_1\cos(\theta_1) - \tilde{n}_2\cos(\theta_2)}{\tilde{n}_1\cos(\theta_1) + \tilde{n}_2\cos(\theta_2)}\right)^2$$ general case

, etc.

Reflection

π-polarization (TM)
(Transversal Magnetic field)





Excited dipoles cannot reemit light in the direction parallel to the dipole vector $\Rightarrow$ no reflected light if,

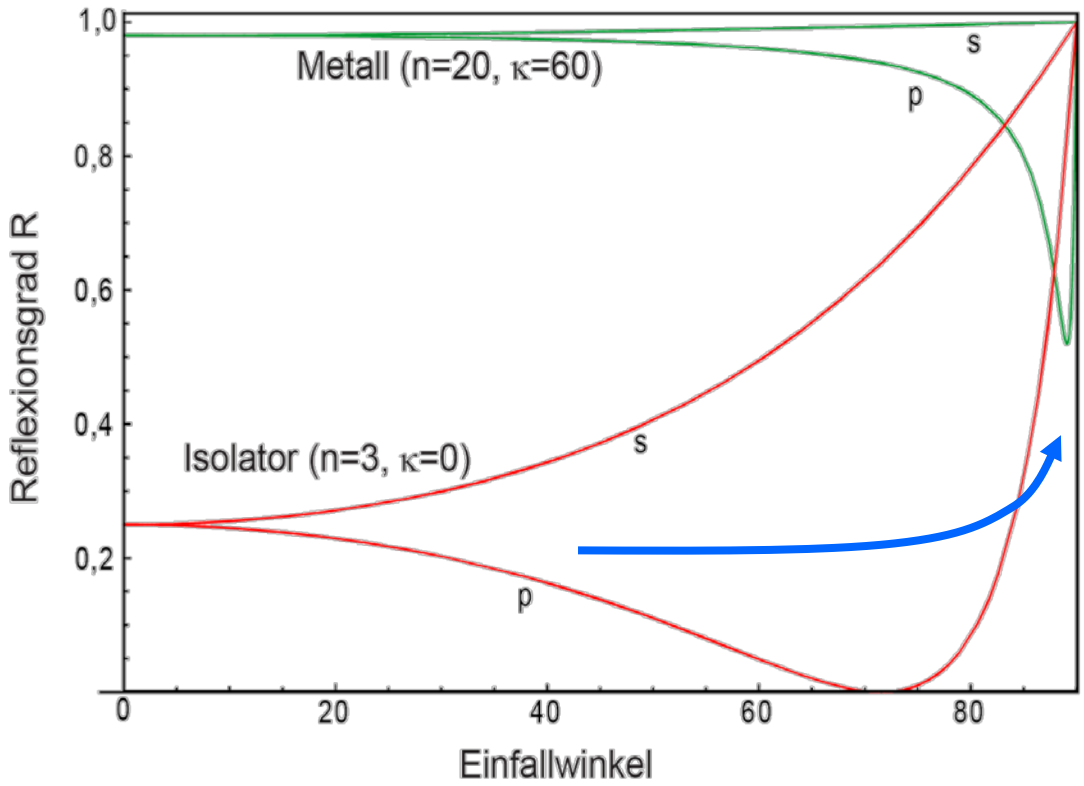
$$\theta_1 + \theta_2 = 90°$$

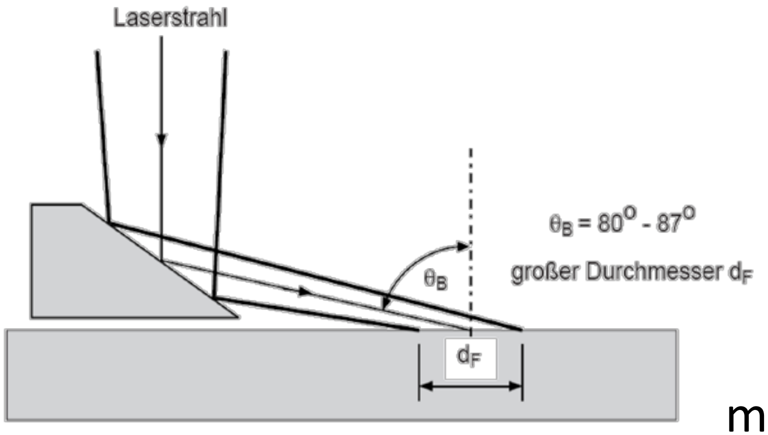
$$\theta_1^B = \arctan \frac{n_2}{n_1}$$

Brewster Angle

Reflection

$$\theta_1^B = \arctan \frac{n_2}{n_1}$$





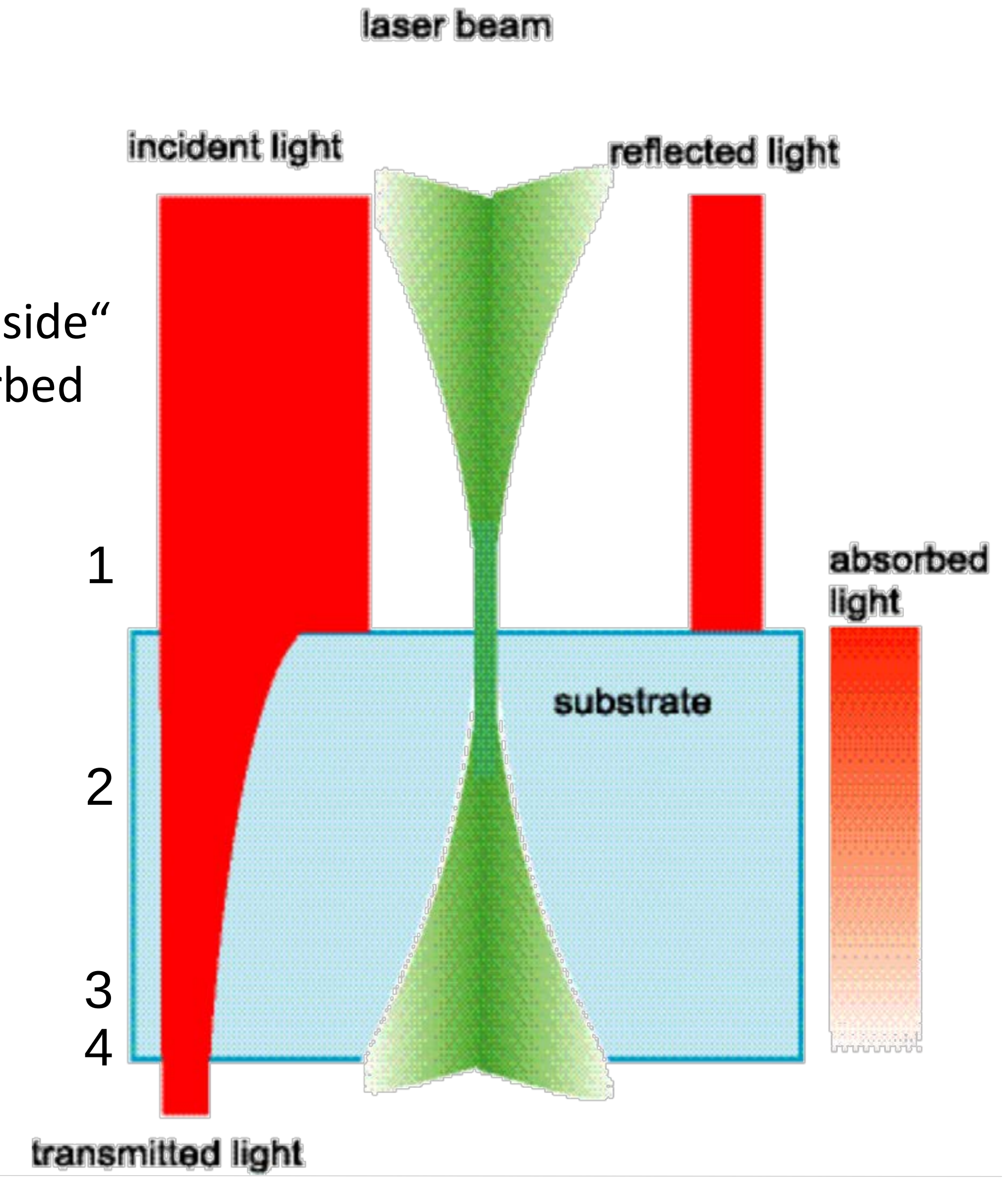
machining under Brewster angle
(increase absorption for metals)

Laser light material interaction

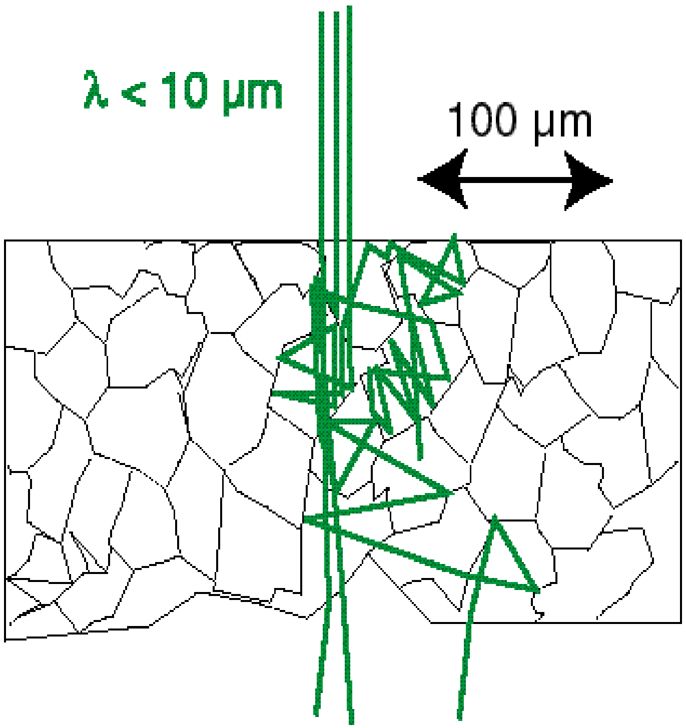
1. Incident -> Reflected + “Passing inside“
2. „Passing inside“ -> partialily absorbed

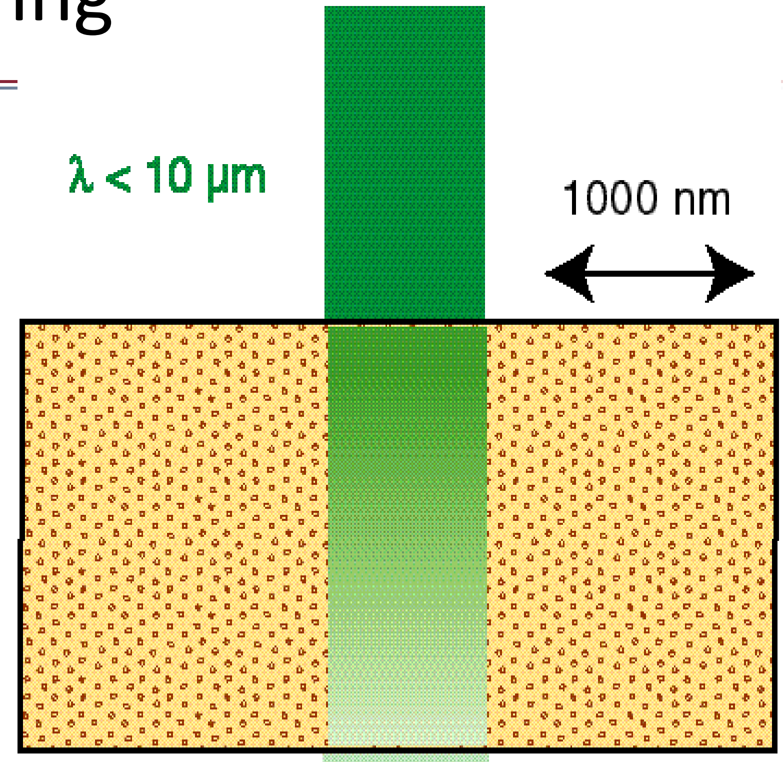
$$I = I_0 e^{-\alpha \cdot l}$$

3. Reflection from the back side
4. The rest is transmitted



Scattering



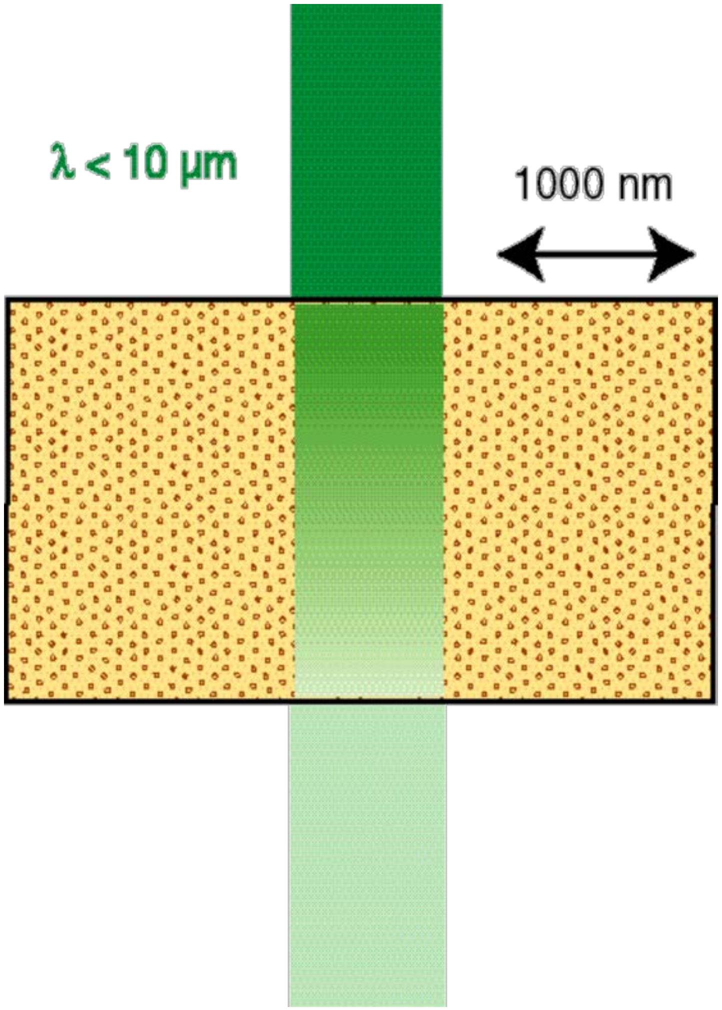


$\lambda \sim d \Rightarrow$ Mie Scattering

$\lambda >> d \Rightarrow$ Rayleigh Scattering

Effective Medium Approximation

ε_1 - inclusion dielectric constant
ε_0 - matrix medium dielectric constant
ε_{eff} – effective dielectric constant
η_1 – volume fraction of inclusions



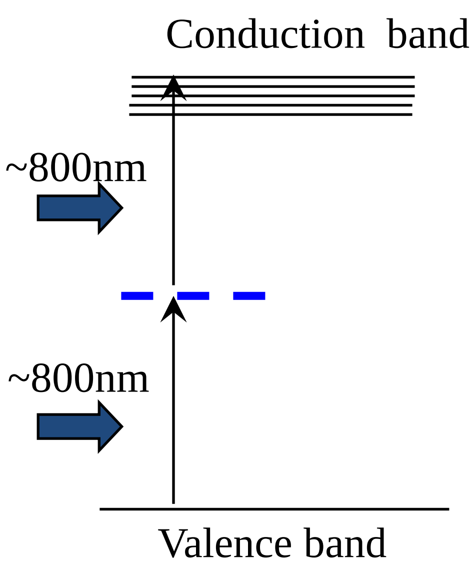
Maxwell-Garnett Equation

$$\frac{\varepsilon_{eff}-\varepsilon_0}{\varepsilon_{eff}+2\varepsilon_0}=\eta_1\frac{\varepsilon_1-\varepsilon_0}{\varepsilon_1+2\varepsilon_0}$$

formula valid only for low concentraitions η_1 and small difference between ε_1 and ε_0

Multiphoton absorption

Two photon absorption



linear absorption $\mathbf{dI} = -\alpha \cdot I \cdot dz$

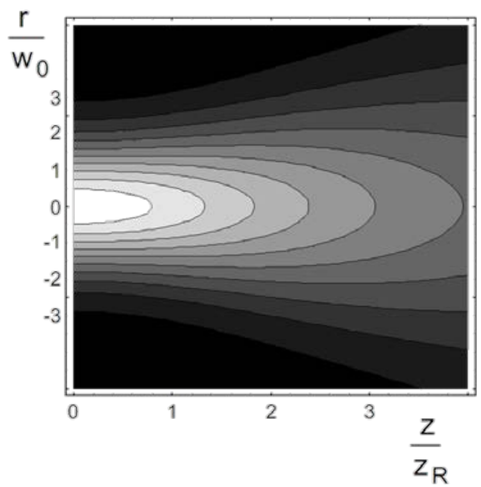
non-linear absorption $\mathbf{dI} = -\alpha(I) \cdot I \cdot dz$

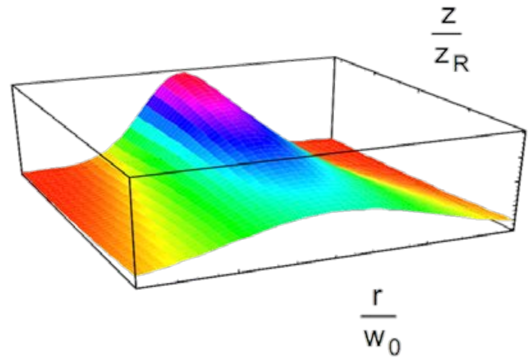
two-photon absorption $\mathbf{dI} = -\alpha^{non-linar} \cdot I^2 \cdot dz$

$(\alpha(I) = \alpha^{non-linear} I)$

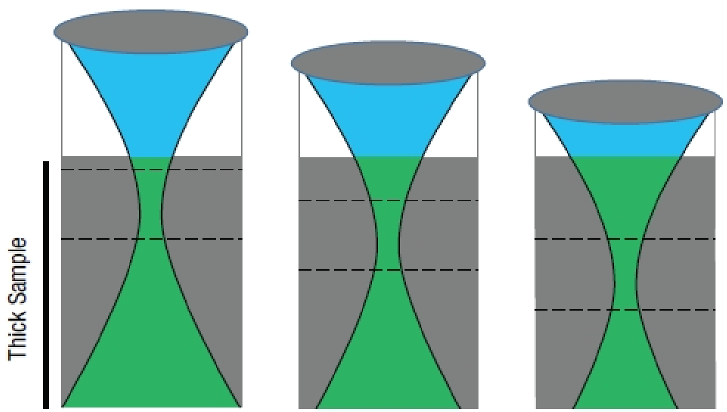
in order to get many photons in the same place at the same time high intensity is required $\Rightarrow$ short-pulse lasers (ps, fs)

Localisation of Multiphoton Absorption

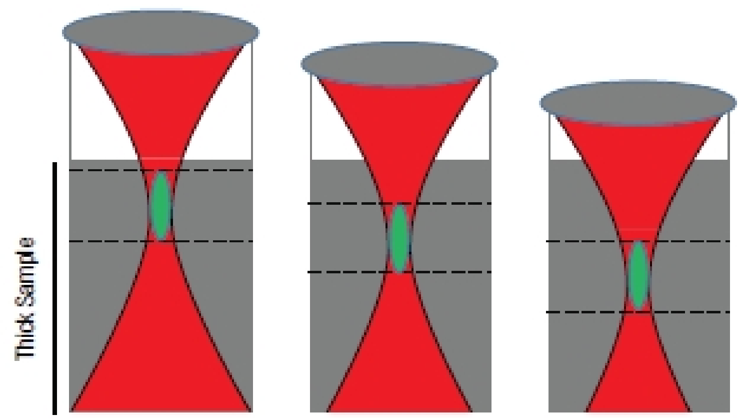




Linear (single photon) absorption

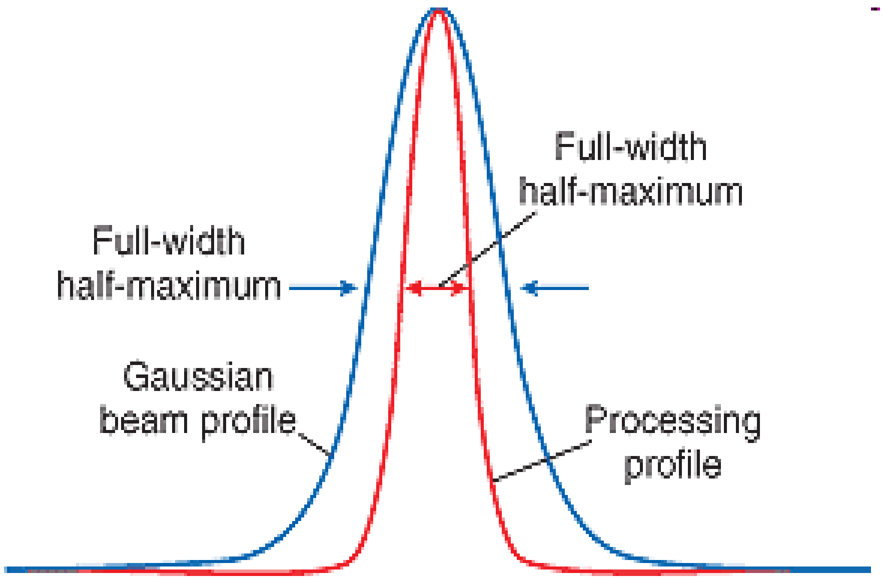


Multiphoton absorption



interaction region is also localised in Z

Lateral Resolution



better localization
due to ~I^n